Tiling Learning Task

Unit 2

Course

Mathematics I: Algebra, Geometry, Statistics

Overview

The launching task for the unit, Tiling, provides a guided discovery for the following:
For each positive integer k, the expression 2k – 1 give the k –th odd positive integer.
For each positive integer k, the expression 2k give the k –th even positive integer.
For each positive integer k, the expression k(k + 1) gives a positive integer because one of k or k + 1 is even.
The k-th triangular number, denoted by Tk, is defined to be the sum of the first k positive integer.

$$T_k = \frac{k(k+1)}{2}$$ so the sum of the first k positive integers is equal to $\frac{k(k+1)}{2}$.

Every square number is the sum of two consecutive triangular numbers; in particular, if Sk denotes the k-th square number, k^2 *then* $S_k = T_{k-1} + T_{k-1} + T_k$.

As the task progresses, students get lots of practice writing algebraic expressions to reflect geometric relationships they have observed. They also write several equations. They write equations that provide a computational formula, such as the equation in (5) above, but they also write equations that express equivalence between different expressions, such as the equation in (6). To be successful in working with the rules for adding, subtracting, multiplying, dividing, and factoring algebraic expressions, students develop this latter understanding of the use of equals. The developmental focus of the task is work with equivalent algebraic expressions. The concept of solving quadratic equations is also foreshadowed. Thus, it can serve to launch a unit focused on traditional algebraic manipulation with polynomials, rational expressions, and radical expressions and an introduction to solving quadratic equations using factoring and taking square roots.

Key Standards

MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

Possible Materials

- Square Items to model tile patterns (crackers, cheez-its, math tiles, colored tiles, samples from local flooring store)
- overhead transparencies
- chart paper
- giant post-its
- bulletin board paper
- chart paper

Task

Latasha and Mario are high school juniors who worked as counselors at a day camp last summer. One of the art projects for the campers involved making designs from colored one-square-inch tiles. As the students worked enthusiastically making their designs, Mario noticed one student making a diamond-shaped design and wondered how big a design, with the same pattern, that could be made if all 5000 tiles available were used. Later in the afternoon, as he and Latasha were putting away materials after the children had left, he mentioned the idea to Latasha. She replied that she saw an interesting design too and wondered if he were talking about the same design. At this point, they stopped cleaning up and got out the tiles to show each other the designs they had in mind.

Mario presented the design that interested him as a sequence of figures as follows:

Figure 1
Figure 2
Figure 3
Figure 4
Figure 5
Figure 6

1.
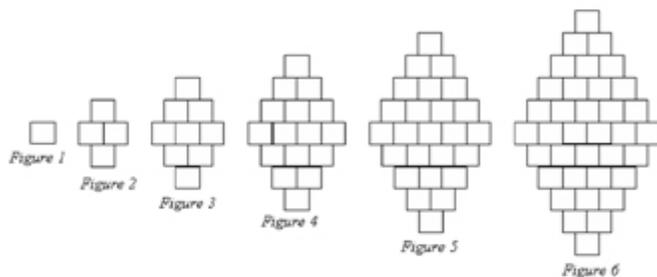
To make sure that you understand the design that was of interest to Mario, answer the following questions.

Comments:
In this set of questions, students are asked to find two patterns: the pattern for the number of rows in each figure and the pattern for the total number of tiles in each figure. They should quickly notice that the number of rows produces the sequence of odd numbers and the number of tiles produces the sequence of square numbers. It is important that students explain these number patterns in terms of the figures and not just accept the pattern without connecting it to the features of the figure. That is, it is important that students use this activity as a chance for sound mathematical reasoning rather than assume that the pattern continues

a) How many rows of tiles are in each of Mario's figures?

Solution:

Figure number	1	2	3	4	5	6
Number of rows	1	3	5	7	9	11

b) What pattern do you observe that relates the number of rows to the figure number? Explain in a sentence.

Solution:
Some likely descriptions are listed, but students may find others that are equivalent.
(i) The number of rows is the figure number plus one less than the figure number.
(ii) The number of rows is one less than twice the figure number.
(iii) Thinking of the figures as gaining two rows (in the middle) each time, the number of rows is 1 plus 2 times one less than the figure number.

c) Use this pattern to predict the number rows in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.

Solution:
Predictions: 23 rows in Figure 12; 93 rows in Figure 47; 221 rows in Figure 111.

d) Write an algebraic expression for the number of rows in Figure k. Explain why your pattern will always give the correct number of rows in Figure k. Can your expression be simplified? If so, simplify it.

Solution:
The expression given by each student will depend on the explanation of the relationship.
(i) k + (k – 1) which simplifies to 2k – 1; (ii) 2k – 1; (iii) 1 + 2(k – 1) which simplifies to 2k – 1. Students should describe the pattern for Figure k in terms of the relationship between the figure number and the number of rows. For example, for description (i) of the pattern: In Figure k, we have one tile in row 1, two tiles in row 2, and so forth until there are k tiles in row k. Then we have k – 1 more rows because the next row has k – 1 tiles and each row after that has one less tile until the last row has only one tile.

e) What is the total number of tiles in each figure above?

Solution:
1 tile in Figure 1, 4 tiles in Figure 2, 9 tiles in Figure 3, 16 tiles in Figure 4, 25 tiles in Figure 5, 36 tiles in Figure 6

f) What pattern do you observe that relates the total number of tiles to the figure number? Explain in a sentence.

Solution:
The number of tiles is the square of the figure number.

g) Use this pattern to predict the total number of tiles in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.

Solution:
Predictions: 144 tiles in Figure 12; 2209 in Figure 47; 12321 tiles in Figure 111.

h) Write an algebraic expression for the total number of tiles in Figure k. Explain why your pattern will always give the correct total number of tiles in Figure k.

Solution:
k^2 tiles in Figure k because the tiles can be rearranged to make k rows with k tiles in each row.

Step 1   Step 2   Step 3   Step 4
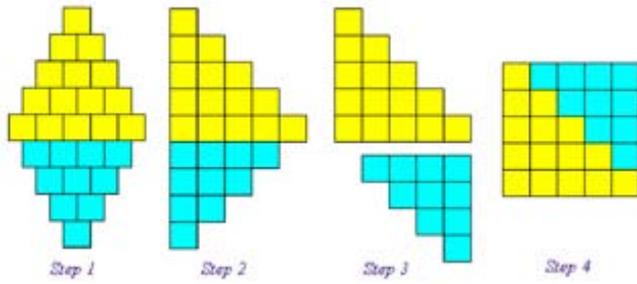
When Latasha saw Mario's figures, she realized that the pattern Mario had in mind was very similar to the one that caught her eye, but not quite the same. Latasha pushed each of Mario's designs apart and added some tiles in the middle to make the following sequence of figures.

Figure 1

Figure 2

Figure 3

Figure 4
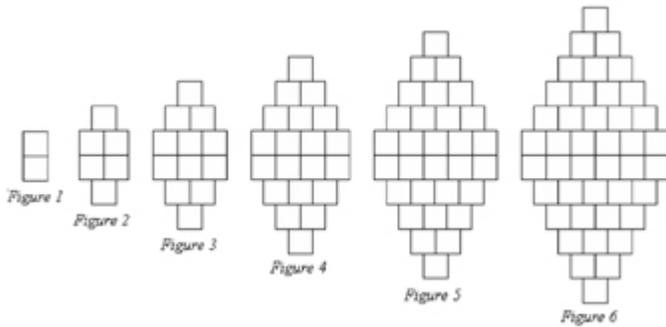
Figure 5

Figure 6

2.
Answer the following questions for Latasha's figures.

Comments: This set of questions parallels the first. Again, it is important that students explain these number patterns in terms of the figures and not just accept the pattern without connecting it to the features of the figure.

a) How many rows of tiles are in each of the figures above?

Solution:

Figure number	1	2	3	4	5	6
Number of rows	2	4	6	8	10	12

b) What pattern do you observe that relates the number of rows to the figure number? Explain in a sentence.

Solution:
Some likely descriptions are listed, but students may find others that are equivalent.
(i) The number of rows is the figure number plus the figure number.
(ii) The number of rows is twice the figure number.
(iii) Thinking of the figures as gaining two rows (in the middle) each time, the number of rows is 2 plus 2 times one less than the figure number.

c) Use this pattern to predict the number rows in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.

Solution:
Predictions: 24 rows in Figure 12; 94 rows in Figure 47; 222 rows in Figure 111.

d) Write an algebraic expression for the number of rows in Figure k. Explain why your pattern will always give the correct number of rows in Figure k. Can your expression be simplified? If so, simplify it.

Comment:
The expression given by each student will depend on the explanation of the relationship.

Solution:
$k + k$ which simplifies to $2k$;
(ii) $2k$;
(iii) $2 + 2(k - 1)$ which simplifies to $2k$.

Comments:
Students should describe the pattern for Figure k in terms of the relationship between the figure number and the number of rows. For example, the following explanation matches description (i) of the pattern: In Figure k, we have one tile in row 1, two tiles in row 2, and so forth until there are k tiles in row k.. Then we have k more rows because the next row has k tiles and each row after that has one less tile until the last row has only one tile.

e) What is the total number of tiles in each figure above?

Solution:
2 tiles in Figure 1, 6 tiles in Figure 2, 12 tiles in Figure 3, 20 tiles in Figure 4, 30 tiles in Figure 5, 42 tiles in Figure 6

f) What pattern do you observe that relates the total number of tiles to the figure number? Explain in a sentence.

Solution:
The number of tiles is the square of the figure number plus the figure number.

g) Use this pattern to predict the total number of tiles in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.

Solution:
Predictions: 156 tiles in Figure 12; 2256 in Figure 47; 12432 tiles in Figure 111.

h) Write an algebraic expression for the total number of tiles in Figure k. Explain why your pattern will always give the correct total number of tiles in Figure k.

Solution:
Each of Latasha's figures have a row with k tiles inserted in the middle of each of Mario's figures. There were k2 tiles in each of Mario's figures, so there are k2 + k tiles in Latasha's Figure k. Alternate answer (there may be others): There are k(k + 1) tiles in Latasha's Figure k because the tiles can be rearranged to make k rows with k + 1 tiles in each row.

i) Give a geometric reason why the number of tiles in Figure k is always an even number. Look at the algebraic expression you wrote in part d.

Solution:
The number of tiles is an even number because each of Latasha's figures can be divided in the middle to make two parts that have the same number of tiles in each part. So, the number of tiles is always twice the number of tiles in the top half.

j) Give an algebraic explanation of why this expression always gives an even number. [Hint: If your expression is not a product, use the distributive property to rewrite it as a product.]

Solution:
The number of tiles in Figure k can be written as k(k + 1). Since k and k + 1 are two consecutive integers, one of them is even and the other is odd. Thus, the product is always even.

3.
Mario started the discussion with Latasha wondering whether he could make a version of the diamond pattern that interested him that would use all 5000 tiles that they had in the art supplies. What do you think? Explain your answer. If you can use all 5000 tiles, how many rows will the design have? If a similar design cannot be made, what is the largest design that can be made with the 5000 tiles, that is, how many rows will this design have and how many tiles will be used?

Comments: Students should be able to use their understandings of square roots to answer this question, but it is designed to foreshadow the concept of solving quadratic equations because we have information about k2 and we want to find k.

Solution:
No, we cannot make one of Mario's designs using all 5000 tiles. There are a square number of tiles in each of Mario's figures, and 5000 is not a square number. The largest square number less than 5000 is 4900 = 70^2. Note that 71^2 = 5041. So, the largest design we can make using no more than 5000 tiles is a design with 139 rows and 4900 tiles.

4.

What is the largest design in the pattern Latasha liked that can be made with no more than 5000 tiles? How many rows does it have? Does it use all 5000 tiles? Justify your answers.

Comment: This question asks students to reason about the expression for the number of tiles in Latasha's figures and find it's largest value less than 5000. It too foreshadows solving quadratic equations.

Solution:
The largest one of Latasha's figures that can be made with no more than 5000 tiles is the figure for k = 70 because 70^2 + 70 = 4970 < 5000 and the figure for k = 71 would take 71^2 +71 = 5112 tiles.

Let M_1, M_2, M_3, M_4, and so forth represent the sequence of numbers that give the total number of tiles in Mario's sequence of figures.

Let L_1, L_2, L_3, L_4 and so forth represent the sequence of numbers that give the total number of tiles in Latasha's sequence of figures.

Comments: Having students write equations that simply state equivalence is intended to enhance understanding of the concept of equality. Research shows that many students view equal signs as an instruction to "find the answer." This view hinders their ability to appreciate the rules of algebra, which are always stated as equality of different algebraic expressions. The language of sequences is used to review this concept and see it in a different context.

5.

Write an equation that expresses each of the following:

a) the relationship between L_1 and M_1 b) the relationship between L_2 and M_2

c) the relationship between L_3 and M_3 d) the relationship between L_4 and M_4

e) the general relationship between L_k and M_k, where k can represent any positive integer.

Solution:

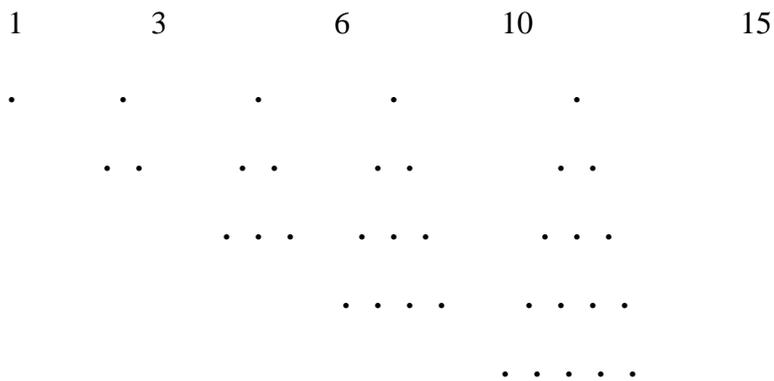   *a) $L_1 = M_1 + 1$*
    b) $L_2 = M_2 + 2$
    c) $L_3 = M_3 + 3$
    d) $L_4 = M_4 + 4$
    e) For any positive integer k, $L_k = M_k + k$.

Triangular numbers are positive integers such that the given number of dots can be arranged in an equilateral triangle. The first few triangular numbers are as follows.

    1   3   6   10   15

Let T_1, T_2, T_3, T_4, and so forth represent the sequence of triangular numbers.

Comment on Question #6: The focus of the following questions is developing an understanding that the k-th triangular number is the sum of the first k positive integers.

6.

a.) Examine the arrangement of dots for T_4. How many dots are in row 1? row 2? row 3? row 4?

Solution:
Row 1 has 1 dot, row 2 has 2 dots, row 3 has 3 dots, row 4 has 4 dots.

b.) Write T_4 as a sum of four positive integers.

Solution: $T_4 = 1 + 2 + 3 + 4$

c.) Write each of T_2, T_3, and T_5 as a sum 2,3, and 5 positive integers, respectively.

Solution:
$$T_2 = 1 + 2; \quad T_3 = 1 + 2 + 3; \quad T_5 = 1 + 2 + 3 + 4 + 5$$
d.) Explain what sum you would need to compute to find T_{12}. Compute T_{12} doing the addition yourself.

Solution:
To find T_{12} you need to find the sum $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12$.
$T_{12} = 78$

e.) Explain what sum you would need to compute to find T_{47}. Use technology to find T_{47}.

Solution:
To find T_{47}, you need to compute the sum of the first 47 positive integers:
$$1 + 2 + 3 + 4 + 5 + \ldots + 45 + 46 + 47. \quad T_{47} = 1128.$$

Comments on Question #7:
This question asks students to find the formula ($\frac{1}{2}$)$k(k + 1)$ for finding the value of the k-th triangular number. Students should find the formula themselves because the beginning parts guide them to see that the triangular arrangements has half as many tiles as the corresponding figure Latasha made. The last parts of the question guide students to see that although the algebraic form of the expression contains a fraction, consideration of all the algebraic information in the formula shows that the calculation will always yield a whole number. A possible extension at this point would be to ask students if it would make sense to consider a 0-th triangular number. If so, what would it be?

When students have completed this question, they should summarize these ideas by writing and understanding the string of equalities: $1 + 2 + 3 + \ldots + k = T_k = \dfrac{k\,(k + 1)}{2}$.

Students should verbalize that they can find the sum of the first k positive integers by multiplying the last integer in the sum, k, by the next integer, k + 1, and dividing the product by 2. For example, to find the sum $1 + 2 + 3 + \ldots + 1000$ we multiply 1000 by 1001 and divide by 2, which is the same as $500(1001) = 500500$.

At this point, students would probably enjoy hearing the story about the story about the famous German mathematician Karl Frederick Gauss when he was a school boy of 10-years old and seeing the proof of the above formula based on Gauss's idea.   The following excerpt from the biographical sketch of Gauss, pp. 1196-1197, Calculus with Analytic Geometry, Fourth Edition, Howard Anton, John Wiley & Sons, 1992, tells the story as follows:
For his elementary education Gauss was enrolled in a squalid school run by a man named Büttner whose main teaching technique was thrashing. Büttner was in the habit of assigning long addition problems which, unknown to his students, were arithmetic progressions that he could sum up using formulas. On the first day Gauss entered the arithmetic class, the students were asked to sum the numbers from 1 to 100. But no sooner had Büttner stated the problem than Gauss turned over his slate and exclaimed in his peasant dialect, "Ligget se'." (Here it lies.) For nearly an hour Büttner

glared at Gauss, who sat with folded hands while his classmates toiled away. When Büttner examined the slates at the end of the period, Gauss's slate contained a single number, 5050 – the only correct solution in the class.

Gauss had realized that he could pair 1 and 100, 2 and 99, 3 and 98, and so forth through pairing 50 and 51 to get 50 pairs of numbers with each pair summing to 101. So the answer would be 5050.

Classic proof of the formula for the sum of the first k integers:
Let $S = 1 + 2 + 3 + \ldots + (k - 2) + (k - 1) + k$. Then, it is also true that

$$S = k + (k - 1) + (k - 2) + \ldots + 3 + 2 + 1.$$

Adding vertically we get the following with k terms on the right.
$$2S = (k + 1) + (k + 1) + (k + 1) + \ldots + (k + 1) + (k + 1) + (k + 1),$$
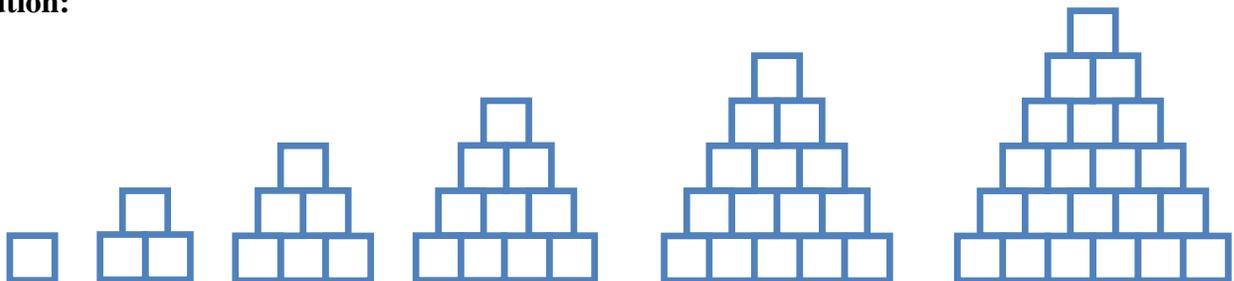
so $2S = k(k + 1)$. Then we divide both sides by 2 to obtain

$$S = \frac{k(k+1)}{2}$$

7. The triangular numbers could also be represented by arrangements of square tiles instead of dots.

   a) Draw the first few figures to represent triangular numbers using square tiles.

Solution:

   b) Compare these figures to Latasha's diamond-shaped figures. Write a sentence comparing the triangular-number figures and Latasha's figures.

Solution:
Each of Latasha's figures consists of two copies of the corresponding triangular number figure, one right-side up and the other upside down.

c) Write an equation using L_k and T_k to express the relationship between the number tiles in the k-th one of Latasha's figures and the k-th triangular number figure.

Solution: $\quad \mathbf{L_k = 2\ T_k,\quad or\quad T_k = (\frac{1}{2})\ L_k}$

d) Check your equation by comparing the numbers L1 and T1, L2 and T2, L3 and T3, L4 and T4, L5 and T5, L12 and T12, and L47 and T47

Solution: $\mathbf{L_1 = 2,\ T_1 = 1;\ L_2 = 6,\ T_2 = 3;\ L_3 = 12,\ T_3 = 6;}$
$\mathbf{L_4 = 20,\ T_4 = 10;\ L_5 = 30,\ T_5 = 15;\ L_{12} = 156,\ T_{12} = 78;\ L_{47} = 2256,\ T_{47} = 1128.}$

e) Write a formula to calculate T_k, the k-th triangular number, without summing the first k positive integers.

Solution:
$$T_k = \frac{1}{2}\ (k^2 + k)\quad \text{or}\quad T_k = \frac{1}{2}\ k\ (k+1)\quad \text{or}\quad T_k = \frac{k^2 + k}{2}\quad \text{or}\quad T_k = \frac{k\ (k+1)}{2}$$

f) Check your formula by using it to calculate T_1, T_2, T_3, T_4, T_5, T_{12}, and T_{47}.
Solution:

$$T_1 = \frac{1^2 + 1}{2} = \frac{2}{2} = 1,\quad T_2 = \frac{2^2 + 2}{2} = \frac{6}{2} = 3,\quad T_3 = \frac{3^2 + 3}{2} = \frac{12}{2} = 6,\quad T_4 = \frac{4^2 + 4}{2} = \frac{20}{2} = 10,$$

$$T_5 = \frac{5^2 + 5}{2} = \frac{30}{2} = 15,\quad T_{12} = \frac{12^2 + 12}{2} = \frac{156}{2} = 78,\quad T_{47} = \frac{47^2 + 47}{2} = \frac{2256}{2} = 1128$$

g) Give a geometric reason why your formula always gives a whole number.

Solution:
The formula gives the number of tiles in the triangular arrangement of tiles, and there is a whole number of tiles in the arrangement.

h) Give an algebraic reason why the formula must always give a whole number.

Solution:
In Question 2, parts i) and j), we saw that $k^2 + k = k\ (k + 1)$ is always an even number. So, half of it is always a whole number.

i)   What sum does your formula calculate?

Solution:
The formula for the k-th triangular number gives a way to calculate the sum of the first k integers, that is, the sum $1 + 2 + 3 + . . . + k$.

Comments on Question # 8:

This last question guides students to see that each square number is the sum of two consecutive triangular numbers. The resulting algebraic statement, $S_k = T_{k-1} + T_k$ (where S_k is the k-th square number) should be verified algebraically at the formula level; that is, students should verify that

$$k^2 = \frac{(k-1)\,k}{2} + \frac{k\,(k+1)}{2}\;.$$

8.

a) Compare the triangular-number figures to Mario's figures. Write a sentence comparing the triangular-number figures and Mario's figures.

Solution:
Each of Mario's figures is a combination of two triangular number figures. Mario's Figure k is the triangular figure for k combined with the triangular figure for k – 1. One of these is right-side-up and the other is up-side-down.

b) Write an equation using M_k and T_k to express the relationship between the number tiles in the k-th one of Mario's figures and the k-th triangular number figure.

Solution: $M_k = T_{k-1} + T_k$

c) What is another name for the sequence M_1, M_2, M_3, M_4, . . . ?

Solution: The sequence of square numbers 1, 4, 9, 16, 25, 36, ...

d) Write a sentence expressing the relationship expressed in part b); use the familiar name for the numbers in the sequence M_1, M_2, M_3, M_4, . . .

Solution: If S_k is the k-th square number, then $S_k = T_{k-1} + T_k$.

Sample Lesson Plans: Tiling Learning Task
Unit 2

Task: Tiling Learning Task Parts #1-#2   **Day:** 1 of 6

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

        a. Simplify algebraic and numeric expressions involving square root.

        b. Perform operations with square roots.

    MM1A3. Students will solve simple equations.

        a. Solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$, by using factorization and finding square roots where applicable.

**Essential Question:** How can I use quadratic expressions to model patterns?

Suggested Materials:

Square items to model tile patterns (crackers, cheez-its, math tiles, colored tiles, samples from local flooring store)

Warm-up:

The diagram at the right shows a sequence of gray and white squares each layered under the previous one.

   a. Explain how the sequence 1, 3, 5, 7, … is related to the area of these squares.

   b. Write a recursive formula that gives the sequence 1, 3, 5, 7, …

   c. What is the 20th number in this sequence?

   d. The first term in the sequence is 1, and the second is 3. What is the 95th?
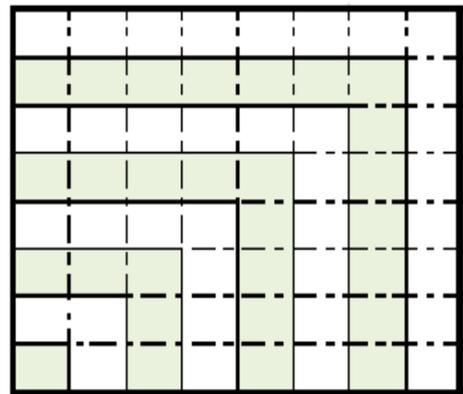
1 3 5 7

Opening/Mini-Lesson:

Discussion of solution to warm-up, identifying linear patterns, and listening for any misconceptions.

Task Time:

15-20 min for Warm-up and opening; 30-35 min for task

> **Watch for:** rows v. columns; using blocks per row, not lines; using explicit formulas instead of recursive; #2 i: students might not understand that "geometric" means make or show a figure

> **Guiding Questions:** How do you use your figure number to produce the number of rows? How can you best organize your information for figure number, number of rows, and number of tiles? How can we transform a recursive formula to an explicit formula? What other figures can you form out of the tiles?

Summary/Closing:

How do the expressions differ between the number of rows and the number of tiles? Geometrically, part 1 produces a square and part 2 produces a rectangle such that the area of the figures will produce the quadratics that models them. See the teacher notes.

Homework:

This 4 x 4 grid contains squares of different sizes. *(Note to teacher: Make sure students understand how to find the 'different sized' squares in the grid.)*

1. How many of each size squares are there? Include overlapping squares.
2. How many total squares would a 3 x 3 grid contain?
3. How many total squares would a 2 x 2 grid contain?
4. How many total squares would a 1 x 1 grid contain?
5. Find a pattern to determine how many squares an n x n grid contains. Use your pattern to predict the number of squares in a 5 x 5 grid.

Suggested Acceleration/Preview for Math Support:

Be sure that students can: recognize the patterns, formulize the patterns, review finite differences, geometric representation of monomials by binomials

Sample Lesson Plan

Unit 2

Task: Tiling Learning Task Parts #3-#4   **Day:** 2 of 6

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

        a. Simplify algebraic and numeric expressions involving square root.

        b. Perform operations with square roots.

    MM1A3. Students will solve simple equations.

        a. Solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$, by using factorization and finding square roots where applicable.

Essential Question: How can I use quadratic expressions to model patterns?

Suggested Materials:

Square Items to model tile patterns (crackers, cheez-its, math tiles, colored tiles, samples from local flooring store), overhead transparencies, chart paper, giant post-its, and bulletin board paper

Warm-up:

  Square numbers

1. I have a square with integer side lengths, what are some of the possible areas of the square?
2. Wh ich one of these could **not** be the area of a square with integer side lengths? Explain why.

    A. 1   B. 24   C. 36   D.64   E. 100

Opening/Mini-Lesson:

Discuss solution to warm-up. Discuss homework solutions-quadratic homework problem leading into the tasks for the day.

Task Time:

Allow 5-10 min for Warm-up and opening; 45-50 min for task (finishing up day 1 if needed, completing tasks #3 and #4, and preparing for presentations on day 3.)

Watch for: Students using correct mathematical language as they prepare for their presentations; improvements from previous presentations; encouraging students to include their own group's efforts; group presentations should be diverse and not all groups will present every time.

Guiding Questions: Can your group present this in a different way? Is your answer reasonable? Do your methods always work? If a group finishes early, some possible extensions are 'Is there a pattern that would use all 5000 tiles?'.

Summary/Closing:

Use a ticket out of the door as a formative assessment:

Homework:

Basic skills practice as needed (square numbers, square roots, simple quadratics).

Suggested Acceleration/Preview for Math Support:

Be sure students know what square numbers are and how to take square roots.

Sample Lesson Plan

Unit: 2

Task: Tiling Learning Task   **Day:** 3 of 6

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

        a. Simplify algebraic and numeric expressions involving square root.

        b. Perform operations with square roots.

   MM1A3. Students will solve simple equations.

        a. Solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$, by using factorization and finding square roots where applicable.

Essential Question: How can I use quadratic expressions to model patterns?

Suggested Materials:

Overheads, overhead transparencies, chart paper, giant post-its, bulletin board paper

Warm-up:

None

Opening/Mini-Lesson:

10 minutes to confer with group members and be ready to present.

Task Time:

45 min for presentations

> **Watch for**: Group presentations should be diverse and not all groups will present every time. Choose the groups and their presentations based on how they addressed the questions and to insure multiple ways are demonstrated. Any specific insights or discussions that occurred should be shared during presentations.

> **Guiding Questions:** Make use of planted questions; Evaluative questions: What made you think to do it that way? Why did you choose this method or representation? Is there another way to explain it? How is your solution different or the same as a previous group?

Summary/Closing: Ticket Out the Door:

Is there a method you would now use instead of yours?

Which of these formulas would be the best representation?

Homework:

Find a pattern that describes the top half of Latasha's design.

Suggested Acceleration/Preview for Math Support:

Continue to address same issues as previous days and make sure students have complete understanding of parts 1-4 of this task.

Sample Lesson Plan

Unit: 2

Task: Tiling Learning Task #5-#7   **Day:** 4 of 6

Standard(s):   MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

a. Simplify algebraic and numeric expressions involving square root.

b. Perform operations with square roots.

MM1A3. Students will solve simple equations.

a. Solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$, by using factorization and finding square roots where applicable.

Essential Question: How can I use quadratic expressions to model patterns?

Suggested Materials:

 Graph paper, isometric dot paper

Warm-up:

Return ticket out the door to students with comments and suggestions written on them, have any class discussion needed to clear up misconceptions shown on the tickets.

Opening/Mini-Lesson: None

Task Time:

 For this day, allow the students to work with partners for 5 minutes to consider possible solutions to #5, and then discuss with class (Think, Pair, Share). Repeat for #6 and #7, adjust time as needed. #7 may need to be broken into smaller sections.

Watch for: Proper subscripts, Students with difficulty at part e) may need to refer back to geometric figures in task #1 and #2.

 Guiding Questions:

#5: Explain how you know the pattern. How do you generalize the formula? Is that the only possibility for the formula? Depending on the formula, what is the independent and dependent variable?

#6: Can you find a short cut for adding up consecutive integers? It is okay if the answer is no. Proceed to #7. Be sure the students recognize how to get the kth triangular number, you need to take the T-sub-k-1 number and add k.

#7: What comparisons can you make from tasks #1 and #2? At part d), check for understanding with teacher before proceeding to part e). What connections can you make from tasks #1 and #2 in relation to this new pattern? Is there a way to write L-sub-k only using k?

Summary/Closing:

Answer student questions.

Homework:

Skills practice – Create assignment if practice on previous skills is needed. If pre-assessment determines this is not necessary, revise appropriately.

Suggested Acceleration/Preview for Math Support: Make sure students are competent is using subscript notation.

Sample Lesson Plan

Unit: 2

Task: Tiling learning Task #8   **Day:** 5 of 6

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

a. Simplify algebraic and numeric expressions involving square root.

b. Perform operations with square roots.

MM1A3. Students will solve simple equations.

a. Solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$, by using factorization and finding square roots where applicable.

Essential Question: How can I use quadratic expressions to model patterns?

Suggested Materials:

Square Items to model tile patterns (crackers, cheez-its, math tiles, colored tiles, samples from local flooring store)

Warm-up:

None

Opening/Mini-Lesson:

Revisit part 6 (or something like it in preparation of #8 part b). Fibonacci sequence, ask for the pattern and use appropriate notation.

Task Time:

Teacher Guided Lesson for Task #8

> **Watch for:** For part b, refer back to task #6; make sure the students are thinking recursively; look at solutions to part 1. h, – *geometrically* see that square numbers are the sum of consecutive triangular numbers.
>
> G**uiding Questions**: See above.

Summary/Closing:

Formative assessment:

Suggested Acceleration/Preview for Math Support:

Review subscript notation once again, paying special attention to the k-th term and how to work with algebraic expressions involving these subscripts.

Sample Lesson Plan

Unit: 2

Task: Tiling Learning Task   **Day:** 6 of 6

Standard(s): MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

> a. Simplify algebraic and numeric expressions involving square root.
>
> b. Perform operations with square roots.

> **MM1A3. Students will solve simple equations.**

> a. Solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$, by using factorization and finding square roots where applicable.

Essential Question: How can I use quadratic expressions to model patterns?

Suggested Materials:

Access to any manipulative used in class.

Warm-up:

Review formative assessment

Task Time:

Review for 15 min:

Quiz for 30-35 min:

Quiz using rectangular numbers, with additional basic skills questions.