Use of This Manual

This training program was developed by the Georgia Department of Education as part of a series of professional development opportunities to help teachers increase student achievement through the use of the Georgia Performance Standards.

For more information on this or other GPS training, you may go to the math webpage through the Georgia Department of Education website under Curriculum and Instruction or use the direct link http://www.gadoe.org/ci_services.aspx?PageReq=ClServMath.

Training Materials

The module materials (guides, presentations, etc.) will be available electronically on http://www.georgiastandards.org under the training tab after all trainings of Days 1, 2 and 3 have occurred. Consult the trainer for availability.
Organization of the Research and Resource Manual

The People Involved

Relevant and Supporting Research

Additional Research

Instructional Resources

  Middle School Learning Tasks

  Algebra Ladder

  High School Learning Tasks

  Article: Never Say Anything a Kid Can Say!

  Article: Tips and Strategies for Co-teaching at the Secondary Level

The Equalizer

Four Parts of a Lesson

Co-Teaching Models Between General and Special Education Teachers

Georgia Student Achievement Pyramid of Interventions

Multiple Representations

Middle School GPS Mathematics Vertical Alignment Charts

High School GPS Mathematics Vertical Alignment Charts

Additional Instructional Resources
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### Partners

- Board of Regents  
  University System of Georgia  
- Center for Education Integrating Science Mathematics and Computing  
- Georgia Institute of Technology  
- Georgia Public Broadcasting  
- Learning and Performance Support Lab  
- University of Georgia
Statement of the Mathematics Advisory Committee

The knowledge and skills that a Georgia high school graduate needs to be successful in college, in the military, or on entry into a high-tech economy require access to the same rigorous curriculum.

Jobs that are characterized as routine in nature, do not involve significant problem solving, and do not involve creativity are jobs that are now being outsourced and will continue to be subject to outsourcing to other countries or to be filled by unskilled labor. Consequently, Georgia students finishing high school need to take responsibility for their own learning, be lifelong learners, be creative, be problem solvers, be adaptable, and have a strong background in mathematics, science, and technology.

Georgia has adapted from the Japanese mathematics curriculum the following characteristics:
1) fewer topics at each grade level
2) more rigor and depth
3) an integrated curriculum
4) a clear, focused path to higher (college) mathematics

The Georgia Performance Standards for mathematics are characterized by the four R’s:
1) Rigor
2) Relevance
3) Relationships, and
4) Reasoning

The Standards strike a balance between concepts, skills, and problem solving. The curriculum provides more depth in concepts than its predecessor, presents real and relevant tasks, and remains strong in computational skills. The Standards offer three sequences (core, regular, and accelerated) that require students to attain a common level of mastery and allow students to move from one sequence to another upon completion of periodic benchmarks.

The Standards require the mastery of the skills, fluency, understanding, and experience needed for economic success regardless of a student’s potential choice of vocation or career. Specifically, they provide a path that enables any student to achieve excellence in careers that demand a high level of mathematical ability.

This revision of the high school GPS results from collaboration by master classroom teachers, district and regional level mathematics leadership, and higher education faculty from both mathematical science departments and colleges of education.

The committee is proud to present the proposed high school Georgia Performance Standards for adoption.
Success in Mathematics at Georgia Tech

Mathematics is an integral part of all science and engineering. A solid mathematical foundation is essential for the successful completion of an undergraduate degree at the Georgia Institute of Technology.

Success does not come from merely covering a shopping list of topics during the K-12 years. In fact, more isn't necessarily better.

AP Calculus credit is welcome, but it is not essential for success at Tech. Far more important, a solid mathematical foundation requires extensive preparation in middle and high school (including the senior year) with focus on mathematical reasoning, problem solving, and an in-depth understanding of algebra, geometry, and functions, including graphing. Incoming Tech students should be able to use mathematics in an applied context; frame the situation as a problem that can be solved using geometric and/or algebraic concepts; and interpret their answers in terms of the context.

The students who succeed at Tech are able to do quick, accurate algebraic computation. They should have experiences in solving mathematical problems that require multiple steps and integration of a variety of topics. They should also be given problems of such difficulty that even the brightest students may take hours or days to solve them. This cannot occur when new topics are introduced at a rapid rate and coverage of material is the primary emphasis.

Prepared by Members of the School of Mathematics, Georgia Tech March 2006
Rena Brakelill
Tom Morley
Enid Steinbert
Yang Wang

A Unit of the University System of Georgia
An Equal Education and Employment Opportunity Institution
How to Bring Our Schools Out of the 20th Century

Sunday, Dec. 10, 2006 by CLAUDIA WALLIS, SONJA STEPTOE

There’s a dark little joke exchanged by educators with a dissident streak: Rip Van Winkle awakens in the 21st century after a hundred-year snooze and is, of course, utterly bewildered by what he sees. Men and women dash about, talking to small metal devices pinned to their ears. Young people sit at home on sofas, moving miniature athletes around on electronic screens. Older folk defy death and disability with metronomes in their chests and with hips made of metal and plastic. Airports, hospitals, shopping malls—every place Rip goes just baffles him. But when he finally walks into a schoolroom, the old man knows exactly where he is. “This is a school,” he declares. “We used to have these back in 1906. Only now the blackboards are green.”

American schools aren’t exactly frozen in time, but considering the pace of change in other areas of life, our public schools tend to feel like throwbacks. Kids spend much of the day as their great-grandparents once did: sitting in rows, listening to teachers lecture, scribbling notes by hand, reading from textbooks that are out of date by the time they are printed. A yawning chasm (with an emphasis on yawning) separates the world inside the schoolhouse from the world outside.

For the past five years, the national conversation on education has focused on reading scores, math tests and closing the “achievement gap” between social classes. This is not a story about that conversation. This is a story about the big public conversation the nation is not having about education, the one that will ultimately determine not merely whether some fraction of our children get “left behind” but also whether an entire generation of kids will fail to make the grade in the global economy because they can’t think their way through abstract problems, work in teams, distinguish good information from bad or speak a language other than English.

This week the conversation will burst onto the front page, when the New Commission on the Skills of the American Workforce, a high-powered, bipartisan assembly of Education Secretaries and business, government and other education leaders releases a blueprint for rethinking American education from pre-K to 12 and beyond to better prepare students to thrive in the global economy. While that report includes some controversial proposals, there is nonetheless a remarkable consensus among educators and business and policy leaders on one key conclusion: we need to bring what we teach and how we teach into the 21st century.

Right now we’re aiming too low. Competency in reading and math—the focus of so much No Child Left Behind (NCLB) testing—is the meager minimum. Scientific and technical skills are, likewise, utterly necessary but insufficient. Today’s
economy demands not only a high-level competence in the traditional academic disciplines but also what might be called 21st century skills. Here's what they are:

Knowing more about the world. Kids are global citizens now, even in small-town America, and they must learn to act that way. Mike Eskew, CEO of UPS, talks about needing workers who are "global trade literate, sensitive to foreign cultures, conversant in different languages"—not exactly strong points in the U.S., where fewer than half of high school students are enrolled in a foreign-language class and where the social-studies curriculum tends to fixate on U.S. history.

Thinking outside the box. Jobs in the new economy--the ones that won't get outsourced or automated-- "put an enormous premium on creative and innovative skills, seeing patterns where other people see only chaos," says Marc Tucker, an author of the skills-commission report and president of the National Center on Education and the Economy. Traditionally that's been an American strength, but schools have become less daring in the back-to-basics climate of NCLB. Kids also must learn to think across disciplines, since that's where most new breakthroughs are made. It's interdisciplinary combinations--design and technology, mathematics and art--"that produce YouTube and Google," says Thomas Friedman, the best-selling author of The World Is Flat.

Becoming smarter about new sources of information. In an age of overflowing information and proliferating media, kids need to rapidly process what's coming at them and distinguish between what's reliable and what isn't. "It's important that students know how to manage it, interpret it, validate it, and how to act on it," says Dell executive Karen Bruett, who serves on the board of the Partnership for 21st Century Skills, a group of corporate and education leaders focused on upgrading American education.

Developing good people skills. EQ, or emotional intelligence, is as important as IQ for success in today's workplace. "Most innovations today involve large teams of people," says former Lockheed Martin CEO Norman Augustine. "We have to emphasize communication skills, the ability to work in teams and with people from different cultures."

Can our public schools, originally designed to educate workers for agrarian life and industrial-age factories, make the necessary shifts? The Skills commission will argue that it's possible only if we add new depth and rigor to our curriculum and standardized exams, redeploy the dollars we spend on education, reshape the teaching force and reorganize who runs the schools. But without waiting for such a revolution, enterprising administrators around the country have begun to update their schools, often with ideas and support from local businesses. The state of Michigan, conceding that it can no longer count on the ailing auto industry to absorb its poorly educated and low-skilled workers, is retooling its high schools, instituting what are among the most rigorous graduation requirements in the nation. Elsewhere, organizations like the Bill and Melinda Gates Foundation, the Carnegie Foundation for the Advancement of Teaching and the Asia Society are pouring money and expertise into model programs to show the way.

What It Means to Be a Global Student

Quick! How many ways can you combine nickels, dimes and pennies to get 20¢? That's the challenge for students in a second-grade math class at Seattle's John Stanford International School, and hands are flying up with answers. The students sit at tables of four manipulating play money. One boy shouts "10 plus 10"; a girl offers "10 plus 5 plus 5," only it sounds like this: "Ju, tasu, go, tasu, go." Down the hall, third-graders are learning to interpret charts and graphs showing how many hours of sleep people need at different ages. "¿Cuantas horas duerme un bebé?" asks the teacher Sabrina Storlie.

This public elementary school has taken the idea of global education and run with it. All students take some classes in either Japanese or Spanish. Other subjects are taught in English, but the content has an international flavor. The school pulls its 393 students from the surrounding highly diverse neighborhood and by lottery from other parts of the city. Generally, its scores on state tests are at or above average, although those exams barely scratch the surface of what Stanford students learn.

Before opening the school seven years ago, principal Karen Kodama surveyed 1,500 business leaders on which languages to teach (plans for Mandarin were dropped for lack of classroom space) and which skills and disciplines. "No. 1 was technology," she recalls. Even first-graders at Stanford begin to use PowerPoint and Internet tools. "Exposure to world cultures was also an important trait cited by the executives," says Kodama, so that instead of circling back to the Pilgrims
and Indians every autumn, children at Stanford do social-studies units on Asia, Africa, Australia, Mexico and South America. Students actively apply the lessons in foreign language and culture by video-conferencing with sister schools in Japan, Africa and Mexico, by exchanging messages, gifts and joining in charity projects. Stanford International shows what's possible for a public elementary school, although it has the rare advantage of support from corporations like Nintendo and Starbucks, which contribute to its $1.7 million-a-year budget. Still, dozens of U.S. school districts have found ways to orient some of their students toward the global economy. Many have opened schools that offer the international baccalaureate (I.B.) program, a rigorous, off-the-shelf curriculum recognized by universities around the world and first introduced in 1968--well before globalization became a buzzword.

To earn an I.B. diploma, students must prove written and spoken proficiency in a second language, write a 4,000-word college-level research paper, complete a real-world service project and pass rigorous oral and written subject exams. Courses offer an international perspective, so even a lesson on the American Revolution will interweave sources from Britain and France with views from the Founding Fathers. "We try to build something we call international mindedness," says Jeffrey Beard, director general of the International Baccalaureate Organization in Geneva, Switzerland. "These are students who can grasp issues across national borders. They have an understanding of nuances and complexity and a balanced approach to problem solving." Despite stringent certification requirements, I.B. schools are growing in the U.S.--from about 350 in 2000 to 682 today. The U.S. Department of Education has a pilot effort to bring the program to more low-income students.

Real Knowledge in the Google Era

Learn the names of all the rivers in South America. That was the assignment given to Deborah Stipek's daughter Meredith in school, and her mom, who's dean of the Stanford University School of Education, was not impressed. "That's silly," Stipek told her daughter. "Tell your teacher that if you need to know anything besides the Amazon, you can look it up on Google." Any number of old-school assignments--memorizing the battles of the Civil War or the periodic table of the elements--now seem faintly absurd. That kind of information, which is poorly retained unless you routinely use it, is available at a keystroke. Still, few would argue that an American child shouldn't learn the causes of the Civil War or understand how the periodic table reflects the atomic structure and properties of the elements. As school critic E.D. Hirsch Jr. points out in his book, The Knowledge Deficit, kids need a substantial fund of information just to make sense of reading materials beyond the grade-school level. Without mastering the fundamental building blocks of math, science or history, complex concepts are impossible.

Many analysts believe that to achieve the right balance between such core knowledge and what educators call "portable skills"--critical thinking, making connections between ideas and knowing how to keep on learning--the U.S. curriculum needs to become more like that of Singapore, Belgium and Sweden, whose students outperform American students on math and science tests. Classes in these countries dwell on key concepts that are taught in depth and in careful sequence, as opposed to a succession of forgettable details so often served in U.S. classrooms. Textbooks and tests support this approach. "Countries from Germany to Singapore have extremely small textbooks that focus on the most powerful and generative ideas," says Roy Pea, co-director of the Stanford Center for Innovations in Learning. These might be the key theorems in math, the laws of thermodynamics in science or the relationship between supply and demand in economics. America's bloated textbooks, by contrast, tend to gallop through a mind-numbing stream of topics and subtopics in an attempt to address a vast range of state standards.

Depth over breadth and the ability to leap across disciplines are exactly what teachers aim for at the Henry Ford Academy, a public charter school in Dearborn, Mich. This fall, 10th-graders in Charles Dershimer's science class began a project that combines concepts from earth science, chemistry, business and design. After reading about Nike's efforts to develop a more environmentally friendly sneaker, students had to choose a consumer product, analyze and explain its environmental impact and then develop a plan for re-engineering it to reduce pollution costs without sacrificing its commercial appeal. Says Dershimer: "It's a challenge for them and for me."

A New Kind of Literacy
The juniors in Bill Stroud's class are riveted by a documentary called Loose Change unspooling on a small TV screen at the Baccalaureate School for Global Education, in urban Astoria, N.Y. The film uses 9/11 footage and interviews with building engineers and Twin Towers survivors to make an oddly compelling if paranoid case that interior explosions unrelated to the impact of the airplanes brought down the World Trade Center on that fateful day. Afterward, the students—an ethnic mix of New Yorkers with their own 9/11 memories—dive into a discussion about the elusive nature of truth.

Raya Harris finds the video more convincing than the official version of the facts. Marisa Reichel objects. "Because of a movie, you are going to change your beliefs?" she demands. "Just because people heard explosions doesn't mean there were explosions. You can say you feel the room spinning, but it isn’t." This kind of discussion about what we know and how we know it is typical of a theory of knowledge class, a required element for an international-baccalaureate diploma. Stroud has posed this question to his class on the blackboard: "If truth is difficult to prove in history, does it follow that all versions are equally acceptable?"

Throughout the year, the class will examine news reports, websites, propaganda, history books, blogs, even pop songs. The goal is to teach kids to be discerning consumers of information and to research, formulate and defend their own views, says Stroud, who is founder and principal of the four-year-old public school, which is located in a repurposed handbag factory. Classes like this, which teach key aspects of information literacy, remain rare in public education, but more and more universities and employers say they are needed as the world grows ever more deluged with information of variable quality.

Last year, in response to demand from colleges, the Educational Testing Service unveiled a new, computer-based exam designed to measure information-and-communication-technology literacy. A pilot study of the test with 6,200 high school seniors and college freshmen found that only half could correctly judge the objectivity of a website. "Kids tend to go to Google and cut and paste a research report together," says Terry Egan, who led the team that developed the new test. "We kind of assumed this generation was so comfortable with technology that they know how to use it for research and deeper thinking," says Egan. "But if they’re not taught these skills, they don’t necessarily pick them up."

Learning 2.0

The chairman of Sun Microsystems was up against one of the most vexing challenges of modern life: a third-grade science project. Scott McNealy had spent hours searching the Web for a lively explanation of electricity that his son could understand. "Finally I found a very nice, animated, educational website showing electrons zooming around and tests after each section. We did this for about an hour and a half and had a ball—a great father-son moment of learning. All of a sudden we ran out of runway because it was a site to help welders, and it then got into welding." For McNealy the experience, three years ago, provided one of life’s aha! moments: "It made me wonder why there isn’t a website where I can just go and have anything I want to learn, K to 12, online, browser based and free."

His solution: draw on the Wikipedia model to create a collection of online courses that can be updated, improved, vetted and built upon by innovative teachers, who, he notes, "are always developing new materials and methods of instruction because they aren’t happy with what they have." And who better to create such a site than McNealy, whose company has led the way in designing open-source computer software? He quickly raised some money, created a nonprofit and—voilà!—Curriki.org made its debut January 2006, and has been growing fast. Some 450 courses are in the works, and about 3,000 people have joined as members. McNealy reports that a teenager in Kuwait has already completed the introductory physics and calculus classes in 18 days.

Curriki, however, isn’t meant to replace going to school but to supplement it and offer courses that may not be available locally. It aims to give teachers classroom-tested content materials and assessments that are livelier and more current and multimedia-based than printed textbooks. Ultimately, it could take the Web 2.0 revolution to school, closing that yawning gap between how kids learn at school and how they do everything else. Educators around the country and overseas are already discussing ways to certify Curriki’s online course work for credit.

Some states are creating their own online courses. "In the 21st century, the ability to be a lifelong learner will, for many people, be dependent on their ability to access and benefit from online learning," says Michael Flanagan, Michigan’s
superintendent of public instruction, which is why Michigan’s new high school graduation requirements, which roll out next year, include completing at least one course online.

A Dose of Reality
Teachers need not fear that they will be made obsolete. They will, however, feel increasing pressure to bring their methods--along with the curriculum--into line with the way the modern world works. That means putting a greater emphasis on teaching kids to collaborate and solve problems in small groups and apply what they've learned in the real world. Besides, research shows that kids learn better that way than with the old chalk-and-talk approach.

At suburban Farmington High in Michigan, the engineering-technology department functions like an engineering firm, with teachers as project managers, a Ford Motor Co. engineer as a consultant and students working in teams. The principles of calculus, physics, chemistry and engineering are taught through activities that fill the hallways with a cacophony of nailing, sawing and chattering. The result: the kids learn to apply academic principles to the real world, think strategically and solve problems.

Such lessons also teach students to show respect for others as well as to be punctual, responsible and work well in teams. Those skills were badly missing in recently hired high school graduates, according to a survey of over 400 human-resource professionals conducted by the Partnership for 21st Century Skills. "Kids don't know how to shake your hand at graduation," says Rudolph Crew, superintendent of the Miami-Dade school system. Deportment, he notes, used to be on the report card. Some of the nation's more forward-thinking schools are bringing it back. It's one part of 21st century education that sleepy old Rip would recognize.

*With reporting by Carolina A. Miranda*
Do the Math: Cognitive Demand Makes a Difference

Extending high expectations to all students in mathematics is a relatively new idea. Even the 1960s movement to improve U.S. mathematics education, which was based on the argument that an excellent scientific education was necessary for a strong economy and national defense, largely was limited to “college-capable” students.

Today, mathematics education faces two major challenges: raising the floor by expanding achievement for all, and lifting the ceiling of achievement to better prepare future leaders in mathematics, as well as in science, engineering, and technology. Although these goals are not mutually exclusive, this Research Points tackles the challenge of ensuring that whole groups of students are not excluded from higher mathematics learning.

In our global economy and democratic society, limiting math education to select students is unacceptable. A recent ACT study provides evidence that college and the workforce require the same levels of readiness in mathematics. One implication: All students require a greater level of “cognitive demand” in mathematics than once was considered appropriate. In other words, high school students need learning experiences in algebra, geometry, data representation, and statistics whether they are planning to enter college or workforce training programs.

The term “cognitive demand” is used in two ways to describe learning opportunities. The first way is linked with curriculum policy and students’ course-taking options — how much math and which courses. The second way relates to how much thinking is called for in the classroom. Routine memorization involves low cognitive demand, no matter how advanced the content. Understanding mathematical concepts involves high cognitive demand, even for basic content. Both types of cognitive demand are associated with student performance on achievement tests, but they are not substitutes for each other.

Course-Taking

Large-scale assessments have found that mathematics achievement can be predicted by the number of mathematics courses taken and the amount of time spent studying advanced mathematics. Generally, these predictors are inter-related.

Course-taking options in the United States are organized according to curricular and ability tracks. Most students are sorted into tracks involving specific course
sequences and, ultimately, different opportunities to learn mathematics. Traditionally, high schools have had three curricular tracks — college preparation, vocational, and general education. The college-preparation track has top status and provides greater opportunity to learn more demanding mathematics.

Although many schools have done away with such three-track sorting, hidden forms of tracking persist. In one common situation, students are divided by perceived ability under the same course label. For example, an algebra course might sort students into fast and slow speeds of learning, so that by the end of the year students in the same class have not had the same opportunity to learn. Another sorting strategy offers different entry points into college-preparatory coursework (e.g., freshman versus junior year). For students who enter the college-preparatory track late in high school, it might be too late to learn enough mathematics to pursue higher-level college courses.

**Signs of Progress**

Despite continued overt or concealed tracking, there has been progress — students who in the past might have been left out of high-demand courses increasingly are being placed in higher-level mathematics. For example, the 1980s saw striking increases in the percentage of African American students earning credits in college-preparatory courses. These increases largely reflect many states’ new standards and graduation requirements for more mathematics credits. Such policies, and their encouraging results, have overlapped with steady upward movement in the percentage of African American students earning undergraduate and master’s degrees in science and engineering.4

In theory, tracking helps all students by providing instruction suited to their ability and learning styles. However, research strongly suggests that not all students are benefiting. Instead, the positive effects of tracking on overall achievement are associated most with a small minority of students assigned to high-status tracks.4 We still need to prepare many more students in elementary and middle school to handle high-demand courses in high school, and we need to figure out how to keep the positive trends moving forward.

**Quality of Mathematical Thinking**

In a review of school impact on the test score gap between African American and white students, Ronald Ferguson concluded that the basic problem is not tracking per se but the expected quality of instruction — the second form of cognitive demand.6

Traditionally, American mathematics teaching has emphasized whole-class lectures with teachers explaining a problem-solving strategy and students passively listening. The lecture usually is followed by students working alone on a large set of problems that reflect the lecture topic.14-16 In contrast, high cognitive demand mathematics programs generally deviate in important ways from the “normal” approaches to mathematics instruction and classroom practice.

The 1989 Trends in International Mathematics and Science Study looked at the ways that mathematics instruction differs among seven countries.17 It found that although effective teaching varies from culture to culture, the key difference between instruction in the United States (the lowest performer in the study) and the other countries was the way teachers and students work on problems as a lesson unfolds.18

While higher achieving countries did not use a larger percentage of high cognitive demand tasks compared to the United States, tasks here rarely were enacted at a high level of cognitive demand. High-performing countries avoided reducing mathematics tasks to mere procedural exercises involving basic computational skills, and they placed greater cognitive demands on students by encouraging them to focus on concepts and connections among these concepts in their problem-solving.

Other research found that in classrooms in which instructional tasks were set up and enacted at high levels of cognitive demand, students did better on measures of reasoning and problem-solving than did students in classrooms in which such tasks were set up at a high level but declined into merely “following the rules,” usually with little understanding.6,8 In successful classrooms, task rigor was maintained when teachers or capable students modeled high-level performance or when teachers pressed for justifications, explanations and meaning through questioning or other feedback.

International comparisons also have shown that some top countries teach fewer concepts in greater depth, while U.S. math curriculum is “a mile wide and an inch deep.”8 To focus the wide scope of topics presented to U.S. students, new curriculum guidelines from the National Council of Teachers of Mathematics.
Two Meanings of Cognitive Demand

High-Level Mathematics Course-Taking

The percentage of African American students earning credits in college-preparatory mathematics courses increased dramatically between 1982 and 1990. These increases reflected state policy changes involving new standards and graduation requirements calling for more mathematics credits. Despite the welcome progress, a word of caution: Merely mandating a narrow curriculum consisting of traditional college-prep mathematics courses will not undo problems endemic to the preK–8 mathematics program. Cognitive demand and instructional quality must be raised both in the lower grades and in high school.

Mathematics Tasks in a Classroom

Mathematical tasks convey messages about what mathematics is and what doing mathematics entails. A typical task passes through three phases. High-demand tasks are the starting point. As these tasks are carried out, teachers must keep students engaged in high-level thinking and reasoning, avoiding the urge to do the hard thinking for students when they struggle with a problem. Teachers should encourage students to use more than one problem-solving strategy, represent the problem in multiple ways, and explain and justify their work. High cognitive demands or thinking processes involved in solving a task can include the use of general procedures connected to underlying concepts and meaning, complex thinking, and reasoning strategies.

African American Graduates Earning Credits in Mathematics Courses (Selected Years)

Mathematics Task

- As presented in instructional materials.
- High-demand tasks address important concepts and call for student thinking, not just rote performance.

Mathematics Task

- As set up by the teacher.
- High-demand tasks can be solved in multiple ways using a variety of representations and fostering mathematical communication.

Mathematics Task

- As created by students under teacher guidance.
- Cognitive demands at this step include using procedures and algorithms with attention to concepts, conjecturing, justifying, explaining, and interpreting.


emphasize key mathematical ideas on which to build deep understanding and connections.

Conclusion

Learning math can be tough. Not learning it is tougher. Many students lack access to higher-level mathematics courses and teaching at all levels of precollege schooling. This is unacceptable in the face of the ever-expanding technical demands posed by higher education and the 21st-century job market. Research reveals that strong academic experience is needed for both college and the workforce. Raising the cognitive demand in the curriculum is necessary for enhancing students’ career prospects.

Recent trends show progress, such as growth in the number of minority students taking higher-level mathematics classes and earning degrees in mathematics. Still, there is much work to be done.

Curriculum policies that limit course options restrict opportunities to learn for traditionally underserved students. This problem is compounded by the sorting of students according to ability within the same mathematics classes and the low quality of some mathematics instruction in elementary and middle schools.

Bringing more advantaged students into higher mathematics study and preparing our future leaders in mathematics and science are not mutually exclusive ends. If we teach math at a higher level of cognitive demand, even in the early grades, we can look forward to a future in which high mathematics achievers better reflect the country’s diverse population. To accomplish this, schools need to be staffed by well-prepared teachers, and high curriculum standards should be a priority. Teaching in high-performing schools requires a learning environment that supports sustained student engagement on both basic skills and cognitively demanding conceptual mathematics tasks.

### Bibliography


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**Research Points**

- **Title:** GPS Training Days 1, 2 and 3 Mathematics 1
- **Issue:** Research and Resource Manual
- **Page:** Page 4

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**What Should Policymakers Do?**

First, embrace high expectations for all students in mathematics. Informed civic engagement and a competitive, global economy demand higher levels of technical skill.

Second, institute curriculum policies that broaden course-taking options for traditionally underserved students. This includes avoiding systems of tracking students that limit their opportunities to learn and delay their exposure to college-preparatory mathematics coursework.

Third, raise cognitive demand in mathematics teaching and learning in both elementary and secondary schools. Elevated thinking processes come into play when students focus on mathematical concepts and connections among those concepts. High cognitive demand is reinforced when teachers maintain the rigor of mathematical tasks, for example, by encouraging students to explain their problem-solving.

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**Research Points**

- **Title:** GPS Training Days 1, 2 and 3 Mathematics 1
- **Issue:** Research and Resource Manual
- **Page:** Page 4
New Study Points to Gap Between U.S. High School Curriculum and College Expectations

There is a gap between what high schools are teaching students in their core college preparatory courses and what colleges want incoming students to know. There has been ongoing dialogue, from many corners, about this misalignment. ACT has now delineated this gap with the results of its latest National Curriculum Survey.

ACT has been conducting surveys of this nature for roughly 30 years. Data from our research has helped establish the most widely recognized definition of college readiness in the United States.

The study, which surveyed high school and college instructors across the country, suggests that colleges generally want incoming students to have in-depth understanding of a selected number of fundamental skills and knowledge, while high schools tend to provide less in-depth instruction of a broader range of skills and topics.

"State learning standards are trying to cover too much ground—more ground than colleges deem necessary—in the limited time they have with students. As a result, key academic skills needed for success in college get short shrift. The problem lies more with the state education standards than with the teachers themselves."

Cynthia B. Schmeiser, ACT's education division president and chief operating officer

College instructors take a dim view of the effectiveness of their state's learning standards. Nearly two-thirds (65%), overall, say their state standards prepare students "poorly" or "very poorly" for college-level work in their subject area, whereas most high school teachers believe state standards prepare students "well" or "very well."
The disparity exists across the curriculum:

- In mathematics, high school teachers tend to give advanced content greater importance than do college instructors. College instructors value rigorous understanding of math fundamentals more.
- In science, high school teachers consistently rate knowledge of specific facts and information as more important than an understanding of science process and inquiry skills. College instructors, in contrast, rate these skills in the opposite way.
- In English and writing, college instructors place more importance on basic grammar and usage skills than do high school teachers. College instructors express frustration that incoming students can’t write a complete sentence.
- In reading, high school and college instructors tend to agree on the relative importance of specific skills. However, instruction of reading skills diminishes in high school.

This gap between what high schools are teaching and what colleges expect is a serious problem that state policymakers and education leaders must work to close. There are a number of states that have initiated a P-16 approach to address the issue of misalignment. Indiana, Kentucky and Michigan have already taken steps to improve the alignment of their learning standards with college expectations. We strongly support their efforts and hope other states will follow suit.

Learn more . . .

Links will open in your browser.
- Full Survey Report
- Policy Implications Report
- Press Release
- Policy Alert Message (this email message)
from *Integrated Mathematics Choices and Challenges*:

“An integrated mathematics program is a holistic mathematics curriculum that-------

- Consists of topics from a wide variety of mathematical fields and blends those topics to emphasize the connections and unity among those fields;
- Emphasizes the relationships among topics within mathematics as well as between mathematics and other disciplines;
- Each year, includes those topics at levels appropriate to students’ abilities;
- Is problem centered and application based;
- Emphasizes problem solving and mathematical reasoning;
- Provides multiple contexts for students to learn mathematics concepts;
- Provides continual reinforcement of concepts through successively expanding treatment of those concepts;
- Makes appropriate use of technology

According to Burkhardt, “Nowhere else in the world would people contemplate the idea of a year of algebra, a year of geometry, another year of algebra, and so on.” The following advantages of integrated curricula are adapted from Burkhardt’s discussion (2001):

- Integrated curricula build essential connections through active processing over an extended period that first consists of weeks as the curriculum points out fundamental links and then ultimately encompasses years as the concepts are used in solving problems across a variety of contexts.
- Integrated curricula help make mathematics more usable by making links with practical contexts that give students opportunities to use their mathematics successfully in increasingly challenging problems.
- Integrated curricula avoid long gaps in learning that result from “year-long chunks of one-flavored curriculum.”
- Integrated curricula allow a balanced curriculum with the flexibility to include newer as well as traditional topics of mathematics and to foster problem solving that spans several aspects of mathematics.
- Integrated curricula support equity because different branches of mathematics, for example, algebra and geometry, favor different learning styles, so an entire school year of one branch puts some students at a greater disadvantage than does a more balanced curriculum that includes several areas of mathematics.”

“Merging areas of mathematics reaches its pinnacle when all mathematics is merged. The idea of breaking down all notions of teaching a particular area of mathematics has had proponents throughout the twentieth century. In 1901, E. H. Moore, the mathematics department chair at the University of Chicago, recommended the traditional subject areas of school mathematics—algebra, geometry, and trigonometry—be unified. In the first decades of this century, a unified curriculum was developed by George W. Myers and later by Ernest Breslich at the laboratory Schools at the University of Chicago. (Myers 1911; Breslich 1915, 1916, 1917; Breslich et al. 1916; Senk 1981).

Since the 1960s, the School Mathematics Project (SMP) and other mathematics projects in England have developed curricula in which topics from many different branches of mathematics—probability and statistics, geometry, algebra, and functions—are found in almost every year. Unifying ideas, such as set and logic and transformation, are found throughout, as are real-world applications.

Beginning in the early 1990s, with support from the National Science Foundation, a number of projects have developed materials for grades 9-12 in which the mathematics is developed around broad problem situations. The ARISE curriculum (Consortium on Mathematics and Its Applications [COMAP] 1998) is organized around units or modules based on such themes as fairness and apportionment, Landsat, and animation, with each module designed to provide about a month’s worth of work in a school year. For instance, of the five units in the first course, “Patterns,” “The Game of Pig,” “Overland Trail,” “The Pit and the Pendulum,” and “Shadows,” only the first and last hint at the mathematics involved. The curriculum of the Systematic Initiative for Montana Mathematics and Science (SIMMS 1996) is similar, except that its units are shorter, being about two to two-and-one-half weeks in duration.

These recent curricula exhibit unifying concepts from applied mathematics, including, most important, mathematical modeling, and also simulation, estimation and approximation. The idea that applied mathematics might supply unifying concepts for school mathematics curricula is relatively recent, but the unity between mathematics and its applications has long been recognized. The greatest mathematicians of all time, from Archimedes through Newton and Euler to Gauss, worked in both pure and applied mathematics and back and forth from one to the other.”

“The mathematics curriculum that we have today in U.S. high schools is, for all its ancient subject matter, an artifact of the twentieth century. I used to assume that it had always been there, dating back perhaps to the ancient Babylonians, until I discovered that it have actually evolved from the decisions made by a committee of ten university academicians, known as the ‘Committee of Ten,’ in the early 1900’s. Their motivation was to prepare students for college mathematics, which then meant calculus. Today, despite a century of the greatest growth in knowledge that mathematics has ever enjoyed, such preparation still means calculus; and so the core curriculum, defying all logic, endures in much the same form as it took when our grandparents went to school. Because the books predated the courses, a natural approach in those early days was to build the curriculum around subjects for which individual primers already existed—algebra, geometry, trigonometry, analytic geometry, and calculus. No book existed about all mathematics in 1900, and hence no template for an integrated course.

Many people tend to forget that obviously no books about all of geometry or algebra or calculus existed either; we had books of selected topics that could serve only as introductions to those subjects. After a century of repetition, those original topics have gradually become definitive, so that many people now assume, for example, that a 900-page textbook entitled Geometry must surely contain everything that is important about geometry. The mere fact that nobody ever planned it that way suggests that this assumption is not likely to be true. Mathematicians should know that it is not even close.

While on the subject of circumstances that nobody ever planned, let me add that no mathematics textbook in 1900 would have contained 900 pages, either. New discoveries and the needs of the workplace have actually prompted some curricular reforms over the years, but, incredibly, they have served only to add more material to what was already one of the most content-laden subjects in the high school curriculum. The bloated textbooks of today consequently contain sections on applied matrix algebra, descriptive statistics, transformational geometry, linear programming, computer programming, game theory, graph theory, Boolean algebra, and optimization—topics that were unheard of forty years ago—whereas only a few of the topics from forty years ago (log tables and trig tables come to mind) have been dropped. The sad effect of adding these topics without regard to their effect on teachers and students has been chaos, with teachers’ picking and choosing topics from textbooks that are far too enormous from which to teach, often omitting, or shortchanging, the very topics that the bloated textbooks were designed to preserve.

All this accumulation leads, I hope, to the conclusion that it is time to rethink the core mathematics curriculum. Defending the current curriculum on the grounds that it has always been the curriculum is not only specious but counterproductive. Defending the current curriculum on the grounds that it represents what mathematics ‘is’ is a trivialization of mathematics. We are teaching an accident of evolution, a collection of topics that began as discrete nineteenth-century primers and accumulated a disjointed bunch of modern topics along the way. We ought to be teaching an integrated twenty-first-century primer that is the product of careful design. Doing so would mean, however, throwing out some non-essential topics that some might consider sacred. It would entail getting back to our mathematical roots, some of which are essential and deep but some of which are, to borrow a term from algebra, extraneous.”

Additional Research

Articles:

NCTM’s Mathematics Education Dialogues series addressed Integrated Mathematics curriculum in its January 2001 edition which is available on the Web site www.nctm.org. The most relevant articles are:

- *Integrated Mathematics? Yes, but Teachers Need Support!* By Bob Trammel
- *The Emperor’s Old Clothes, or How the World Sees It...* by Hugh Burkhardt
- *Integrated Mathematics and High-Stakes Standardized Testing – They Do Go Together!* By Gabe McMillan

Prevost, F. J. “Rethinking How We Teach: Learning Mathematical Pedagogy.” *The Mathematics Teacher*; January 1993; 86, 1; Research Library pg. 75.


Books:


- This book offers research-based evidence on the effectiveness of a standards-based curriculum.


- Offers extensive research on standards-based instruction – a number of articles provide evidence on the effectiveness of the Process Standards (problem solving, mathematical reasoning, communication, making connections, and use of multiple representations).

- Provides a number of articles on the planning, implementing, and assessing integrated mathematics curriculum. The definitions of “integration” vary (some articles address cross curricular integration). Parts 1 and 4 are most applicable for Georgia teachers. In addition, chapter 10 offers a view of integrated college instruction through math modeling that may interest high school teachers.


- Offers 79 research-based strategies that help students and teachers be successful in mathematics – presents research, alignment to NCTM Standards, classroom application, and possible pitfalls.


- Most of the research on integrated curriculum investigates specific curriculum materials. This book provides historical background on reform efforts in mathematics, as well as research on the implementation of various integrated mathematics curricula, looking specifically at the effectiveness of reform (integrated, performance-based) curricula compared to traditional curricula materials.


- The book presents specific cases of mathematics instruction and instructional strategies. Suggestions are made to teachers about how to cultivate a challenging, cognitively rich, and exciting classroom environment that propels students toward a deeper understanding and appreciation of mathematics.


- The goal of this book is to help students develop a deeper understanding of mathematical concepts and methods by engaging them in trying to make sense of problematic tasks in which the mathematics to be learned is embedded. This volume and its companion for PreK-6 furnish the coherence and direction that teachers need to use problem solving to teach mathematics.
Puzzling Percentages

A. If $\frac{3}{4}$ of a cup of juice gives you 60% of your daily value of Vitamin C, what percent of your daily value of Vitamin C will you get by drinking $\frac{1}{2}$ of a cup of juice? Justify your answer using words, pictures, and/or symbols.

B. Is a% of b equal to b% of a? Show how you know.

C. The price of a necklace was first increased by 50% and later decreased by 50%. Is the final price the same as the original price? Justify your answer.
Analyzing Tables

Consider the tables below where the x- and y-values represent two quantities.

A. Do the quantities vary proportionally? Explain how you know.

B. Write a rule for each table in words.

C. Write the rule for each table as an equation.

1. 

<table>
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<th>4</th>
<th>3</th>
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2. 

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3. 

<table>
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<th>2</th>
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<tbody>
<tr>
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<td>$\frac{1}{2}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{5}$</td>
</tr>
</tbody>
</table>
Surprise Birthday Party

Sue decided to give Maria a surprise birthday party. She ordered a large ice cream cake for the party, but is not sure how many people to invite.

A. How will the number of people attending the party affect the portion of cake that each person gets?

B. Is there a relationship between the number of people attending the party and the portion of the cake that each person gets? Explain your answer and illustrate with pictures, a table, and a graph.

C. What is the constant of proportionality and what does it represent in the context of this problem?
A Staircase Task

A same-color staircase is made from Cuisenaire rods. Each time that a rod is added to the staircase, it is offset by the space of a 1 unit rod. The rods that are used to make the staircase are 3 units in length.

A. What is the surface area and volume of a staircase that is 3 units tall?

B. Predict the volume and surface area of a staircase that is 5 units high. Find the actual surface area and volume, and compare them with your answer. Explain any discrepancies that you found.

C. What will the volume and surface area be when you add the hundredth rod?

C. Develop a general method or rule that can be used to determine the volume and surface area for any number of rods. Explain your thinking.
Algebra Ladder

Each color group of statements is a subset of the standards from a single grade level from K to Math 4. Cut and rearrange them into the correct order. Try to do this without using the curriculum.

**Analyse and describe relationships between varying quantities**  
**Solve simple equations**

**Understand relationships between two variables**

Explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.

**Build number patterns using various concrete representations**

*Analyze graphs of polynomial functions of higher degree*  
*Explore logarithmic functions as inverses of exponential functions*  
*Represent and interpret quantities and relationships using mathematical expressions and symbols (=, <, >)*

**Use variables, such as n or x, for unknown quantities in algebraic expressions**  
**Investigate simple algebraic expressions by substituting numbers for the unknown**

**Explore rational functions**

*Use the circle to define the trigonometric functions.*  
*Students will investigate and use the graphs of trigonometric functions*  
*Investigate different types of functions; transformations of functions*

**Understand and apply patterns and rules to describe relationships and solve problems**

*Represent unknowns using symbols, such as _ and Δ*  
*Write and evaluate mathematical expressions using symbols and different values*

**Identify, create, extend and transfer patterns from one representation to another**

**Gather data that can be modeled with a linear function**

*Estimate and determine a line of best fit from a scatter plot*

**Describe and explain a quantitative relationship represented by a formula**

*Use a symbol to represent an unknown and find its value in a number sentence*

**Investigate step and piecewise functions**

*Explore exponential functions.*  
*Analyze quadratic functions.*
Algebra Ladder Solutions

These are in order from K to Math 4.

**Identify, create, extend and transfer patterns from one representation to another**

*Build number patterns using various concrete representations*

*Represent and interpret quantities and relationships using mathematical expressions and symbols (\(=, <, >\))*

*Describe and explain a quantitative relationship represented by a formula*

*Use a symbol to represent an unknown and find its value in a number sentence*

**Understand and apply patterns and rules to describe relationships and solve problems**

*Represent unknowns using symbols, such as _ and \(\Delta\)*

*Write and evaluate mathematical expressions using symbols and different values*

*Use variables, such as \(n\) or \(x\), for unknown quantities in algebraic expressions*

*Investigate simple algebraic expressions by substituting numbers for the unknown*

**Analyze and describe relationships between varying quantities**

*Solve simple equations*

**Understand relationships between two variables**

*Gather data that can be modeled with a linear function*

*Estimate and determine a line of best fit from a scatter plot*

*Explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.*

**Investigate step and piecewise functions**

*Explore exponential functions.*

*Analyze quadratic functions.*

*Analyze graphs of polynomial functions of higher degree*

*Explore logarithmic functions as inverses of exponential functions*

**Explore rational functions**

*Use the circle to define the trigonometric functions.*

*Students will investigate and use the graphs of trigonometric functions*

*Investigate different types of functions; transformations of functions*
Function Learning Task (Function Notation)

1. One vacation when visiting his grandmother, Todd found markings on the inside of a closet door showing the heights of his mother, Julia, and her brothers and sisters on their birthdays growing up. From the markings in the closet, Todd wrote down his mother’s height from ages 2 to 16. His grandmother found the measurements at birth and one year by looking in his mother’s baby book. The data is provided in the table below, with heights rounded to inches.

<table>
<thead>
<tr>
<th>Age (yrs.)</th>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in.)</td>
<td>y = h(x)</td>
<td>21</td>
<td>30</td>
<td>35</td>
<td>39</td>
<td>43</td>
<td>46</td>
<td>48</td>
<td>51</td>
<td>53</td>
<td>55</td>
<td>59</td>
<td>62</td>
<td>64</td>
<td>65</td>
<td>65</td>
<td>66</td>
<td>66</td>
</tr>
</tbody>
</table>

a. Which variable is the independent variable, and which is the dependent variable? Explain your choice.
b. Make a graph of the data.
c. Should you connect the dots on your graph? Explain.
d. Describe how Julia’s height changed as she grew up.
e. How tall was Julia on her 11th birthday? Explain how you can see this in both the graph and the table.
f. What do you think happened to Julia’s height after age 16? Explain. How could you show this on your graph?

2. In Math 1, we will often use function notation to describe relationships between quantities that vary. In function notation, \( h(2) \) means the output value when the input value is 2. In the case of the table above, \( h(2) \) means the y-value when x is 2, which is her height (in inches) at age 2, or 35. Thus, \( h(2) = 35 \). Function notation gives us another way to write about ideas that you began learning in middle school, as shown in the table below.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>At age 2, she was 35 inches tall</td>
<td>Natural language</td>
</tr>
<tr>
<td>When x is 2, y is 35.</td>
<td>Statement about variables</td>
</tr>
<tr>
<td>When the input is 2, the output is 35.</td>
<td>Input-output statement</td>
</tr>
<tr>
<td>( h(2) = 35 ).</td>
<td>Function notation</td>
</tr>
</tbody>
</table>

As you can see, function notation provides shorthand for talking about relationships between variables. With function notation, it is easy to indicate simultaneously the values of both the independent and dependent variables. The notation \( h(x) \) is typically read “\( h \) of \( x \),” though it is helpful to think “\( h \) at \( x \),” so that \( h(2) \) can be interpreted as “height at age 2,” for example.
Note: Function notation looks like an instruction to multiply, but the meaning is very different. To avoid misinterpretation, be sure you know which letters represent functions. For example, if \( g \) represents a function, then \( g(4) \) is not multiplication but rather the value of “\( g \) at 4,” i.e., the output value of the function \( g \) when the input is value is 4.

a. What is \( h(11) \)? What does this mean?
b. When \( x \) is 3, what is \( y \)? Express this fact using function notation.
c. Find an \( x \) so that \( h(x) = 53 \). Explain your method. What does your answer mean?
d. From your graph or your table, estimate \( h(6.5) \). Explain your method. What does your answer mean?
e. Estimate an \( x \) so that \( h(x) = 60 \). Explain your method. What does your answer mean?
f. Describe what happens to \( h(x) \) as \( x \) increases from 0 to 16.
g. What can you say about \( h(x) \) for \( x \) greater than 16?
h. Describe the similarities and differences you see between these questions and the questions in the previous problem.

3. When Peachtree Plains High School opened in 2001, a few teachers and students put on FallFest, featuring contests, games, prizes, and performances by student bands. To raise money for the event, they sold FallFest T-shirts. The event was very well received, and so FallFest has become a tradition. The graph below shows T-shirt sales for each FallFest so far.
a. What are the independent and dependent variables shown in the graph?
b. For which years does the graph provide data?
c. Does it make sense to connect the dots in the graph? Explain.
d. What were the T-shirt sales in 2001? Use function notation to express your result.
e. Find $S(3)$, if possible, and explain what it means or would mean.
f. Find $S(6)$, if possible, and explain what it means or would mean.
g. Find $S(2.4)$, if possible, and explain what it means or would mean.
h. If possible, find a $t$ such that $S(t) = 65$. Explain.
i. If possible, find a $t$ such that $S(t) = 62$. Explain.
j. Describe what happens to $S(t)$ as $t$ increases, beginning at $t = 1$.
k. What can you say about $S(t)$ for values of $t$ greater than 6?

Note: As you have seen above, functions can be described by tables and by graphs. In high school mathematics, functions are often given by formulas, but it is important to remember that not all functions can be described by formulas.

4. Suppose a ball is dropped from a high place, such as the Tower of Pisa. If $y$, measured in meters, is the distance the ball has fallen and $x$, measured in seconds, is the time since the ball was dropped, then $y$ is a function of $x$, and the relationship can be approximated by the formula $y = f(x) = 5x^2$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2$</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>…</td>
</tr>
<tr>
<td>$y = f(x) = 5x^2$</td>
<td>0</td>
<td>5</td>
<td>20</td>
<td>45</td>
<td>100</td>
<td>175</td>
<td>…</td>
<td></td>
</tr>
</tbody>
</table>

a. Fill in the missing values in the table above.
b. Suppose the ball is dropped from a building at least 100 meters high. Measuring from the top of the building, draw a picture indicating the position of the ball at times indicated in your table of values.
c. Draw a graph of $x$ versus $y$ for this situation. Should you connect the dots? Explain.
d. What is the relationship between the picture (part b) and the graph (part d)?
e. You know from experience that the speed of the ball increases as it falls. How can you “see” the increasing speed in your table? How can you “see” the increasing speed in your picture?
f. What is $f(4)$? What does this mean?
g. Estimate $x$ such that $f(x) = 50$. Explain your method. What does it mean?
h. In this context, $y$ is proportional to $x^2$. Explain what that means. How can you see this in the table?
5. Towanda is paid $7 per hour in her part-time job at the local Dairy Stop. Let $t$ be the amount of time that she works, in hours, during the week, and let $P(t)$ be her gross pay (before taxes), in dollars, for the week.
   a. Make a table showing how her gross pay depends upon the amount of time she works during the week.
   b. Make a graph illustrating how her gross pay depends upon the amount of time that she works. Should you connect the dots? Explain.
   c. Write a formula showing how her gross pay depends upon the amount of time that she works.
   d. What is $P(9)$? What does it mean? Explain how you can use the graph, the table, and the formula to compute $P(9)$.
   e. If Towanda works 11 hours and 15 minutes, what will her gross pay be? Show how you know. Express the result using function notation.
   f. If Towanda works 4 hours and 50 minutes, what will her gross pay be? Show how you know. Express the result using function notation.
   g. One week Towanda’s gross pay was $42. How many hours did she work? Show how you know.
   h. Another week Towanda’s gross pay was $57.19. How many hours did she work? Show how you know.
Around the Garden Learning Task (Domain and Range in Context)

1. Claire has decided to plant a rectangular garden in her back yard using 30 pieces of fencing that were given to her by a friend. Each piece of fencing is a panel 1 yard wide and high enough to keep deer out. She wants to determine the possible dimensions of her garden, assuming that she uses all of the fencing and does not cut the panels. She decides to let $x$ be the garden dimension parallel to the back of her house, and to let $y$ be the other dimension. For example, she begins by placing ten panels (10 yards) parallel to the back side of her house and realizes that the other dimension of her garden then will have to be 5 yards, as shown in the picture below. She summarizes this result as follows: When $x = 10$, $y = 5$.

![Diagram of a garden with dimensions 10 yds by 5 yds](image)

a. Explain why $y$ must be 5 when $x$ is 10.
b. Make a table showing the possible lengths and widths for the garden.
c. If $x = 15$, what could $y$ be? Explain why Claire would be unlikely to build such a garden.
d. Can $x$ be 16? What is the maximum possible width? Explain.
e. Write a formula relating the width and length of the garden. For what possible lengths and widths is this formula appropriate? Express these values mathematically.
f. Find the perimeters of each of these possible gardens. What do you notice? Explain why this happens.
g. Make a graph of the possible dimensions of Claire’s garden.
h. What would it mean to connect the dots on your graph? Does connecting the dots make sense for this context? Explain.
i. As the $x$-dimension of the garden increases by 1 yard, what happens to the $y$-dimension? Does it matter what $x$-value you start with? How do you see this in the graph? In the table? In your formula? What makes the dimensions change together in this way?
2. After listing the possible rectangular dimensions of the garden, Claire realizes that she needs to pay attention to the area of the garden, because that is what determines how many plants she will be able to grow.
   a. Will the area of the garden change as the x-dimension changes? Make a prediction, and explain your thinking.
   b. Make a table showing all the possible x-dimensions for the garden and the corresponding areas. (To facilitate your calculations, you might want to include the y-dimensions in your table.)
   c. Make a graph showing the relationship between the x-dimension and the area of the garden. Should you connect the dots? Explain.
   d. Write a formula showing how to compute the area of the garden, given its x-dimension.

3. Because the area of Claire’s garden depends upon the x-dimension, we can say that the area is a function of the x-dimension. Each x-dimension is an input value for the area function, and the resulting area is the corresponding output value. The set of all possible input values for a function is called the domain of the function. The set of all possible output values is called the range of the function.
   a. What is the domain of the area function for this context?
   b. How is the question about domain related to the question about connecting the dots on the graph?
   c. What is the maximum area of the garden, and what are its dimensions? How do you see this in your table? In your graph?
   d. What is the minimum area of the garden, and what are its dimensions? How do you see this in your table? In your graph?
   e. What is the range of the area function for this context? How can you see the range in your table? In your graph?
   f. Claire’s neighbor Javier noticed that the graph is symmetric. Describe the symmetry, specifically indicating the line of symmetry. What about the context causes this symmetry?
   g. As the x-dimension of the garden increases by 1 yard, what happens to the area? Does it matter what x-dimension you start with? How do you see this in the graph? In the table? Explain what you notice.
Later that summer, Claire’s sister-in-law Kenya mentions that she wants to use 30 yards of chain-link fence to build a pen for her pet rabbits. Claire experiences déjà vu and shares the solution to her garden problem but then she realizes that Kenya’s problem is slightly different.

a. In this case, Claire and Kenya agree that the $x$-dimension does not have to be a whole number. Explain.

b. Despite the fact that the rabbit pen would not be useful with an $x$-dimension of 0, it is often valuable, mathematically, to include such “limiting cases,” when possible. Why would a rabbit pen with an $x$-dimension of 0 be called a limiting case in this situation? Why would the shape of the garden be called a “degenerate rectangle”?

c. Are there other limiting cases to consider in this situation? Explain.

d. Make a table for the area versus the $x$-dimension of Kenya’s garden. Be sure to include some non-whole-number $x$-dimensions.

e. Make a graph of the area versus the $x$-dimension of Kenya’s garden. (Note, if the limiting case is *not* included, the point is plotted in the graph as a small “open circle.” If the limiting case is *included* the point is plotted in the graph as a small “closed circle.”)

f. Write a formula showing how to compute the area of the rabbit pen, given its $x$-dimension.

Kenya uses the table, graph, and formula to answer questions about the possibilities for her rabbit pen.

a. Estimate the area of a rabbit pen of with an $x$-dimension of 10 feet (not yards). Explain your reasoning.

b. Estimate the $x$-dimension of a rectangle with an area of 30 square yards. Explain your reasoning.

c. What is the domain of the function relating the area of the rabbit pen to its $x$-dimension? How do you see the domain in the graph?

d. What is the area and what are the dimensions of the pen with maximum area? Explain. And what do you notice about the shape of the pen?

e. What is the range of the area function for the rabbit pen? How can you see the range in the graph?

f. As the $x$-dimension of Kenya’s garden increases, sometimes the area increases and sometimes the area decreases. For what $x$-values does the area increase as $x$ increases? For what $x$-values does the area decrease as $x$ increases? [Note to teachers: Do not use interval notation.]

When using tables and formulas we often look at a function a point or two at a time, but in high school mathematics, it is important to begin to think about “the whole function,” which is to say all of the input-output pairs. A graph of a function is very useful for considering questions about “whole functions,” but keep in mind that a graph might not show all possible input-output pairs.
Two functions are equal (as whole functions) if they have exactly the same input-output pairs. In other words, two functions are equal if they have the same domain and if the output values are the same for each input value in the domain. From a graphical perspective, two functions are equal if their graphs have exactly the same points.

6. Kenya’s rabbit pen and Claire’s garden are very similar in some respects but different in others. These two situations involve different functions, even though the formulas are the same.
   a. If Kenya makes the pen with maximum area, how much more area will her rabbit pen have than Claire’s garden of maximum area? How much area is that in square feet?
   b. What could Claire have done to have built her garden with the same area as the maximum area for Kenya’s rabbit pen? Do you think this would have been worthwhile?
   c. Describe the similarities and differences between Kenya’s rabbit pen problem and Claire’s garden problem. Consider the tables, the graphs, the formulas, and the problem situations. Use the words domain and range in your response.
**Tiling Learning Task** (Exploring Algebraic Expressions)

Latasha and Mario are high school juniors who worked as counselors at a day camp last summer. One of the art projects for the campers involved making designs from colored one-square-inch tiles. As the students worked enthusiastically making their designs, Mario noticed one student making a diamond-shaped design and wondered how big a design, with the same pattern, that could be made if all 5000 tiles available were used. Later in the afternoon, as he and Latasha were putting away materials after the children had left, he mentioned the idea to Latasha. She replied that she saw an interesting design too and wondered if he were talking about the same design. At this point, they stopped cleaning up and got out the tiles to show each other the designs they had in mind.

Mario presented the design that interested him as a sequence of figures as follows:

1. To make sure that you understand the design that was of interest to Mario, answer the following questions.
   a) How many rows of tiles are in each of Mario’s figures?
   b) What pattern do you observe that relates the number of rows to the figure number? Explain in a sentence.
   c) Use this pattern to predict the number rows in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.
   d) Write an algebraic expression for the number of rows in Figure \( k \). Explain why your pattern will always give the correct number of rows in Figure \( k \). Can your expression be simplified? If so, simplify it.
   e) What is the total number of tiles in each figure above?
   f) What pattern do you observe that relates the total number of tiles to the figure number? Explain in a sentence.
   g) Use this pattern to predict the total number of tiles in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.
   h) Write an algebraic expression for the total number of tiles in Figure \( k \). Explain why your pattern will always give the correct total number of tiles in Figure \( k \).
i) Write a summary of your thoughts and conclusions regarding Mario’s sequence of figures.

When Latasha saw Mario’s figures, she realized that the pattern Mario had in mind was very similar to the one that caught her eye, but not quite the same. Latasha pushed each of Mario’s designs apart and added some tiles in the middle to make the following sequence of figures.

![Figures 1 to 6]

2. Answer the following questions for Latasha’s figures.
   a) How many rows of tiles are in each of the figures above?
   b) What pattern do you observe that relates the number of rows to the figure number? Explain in a sentence.
   c) Use this pattern to predict the number rows in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.
   d) Write an algebraic expression for the number of rows in Figure \( k \). Explain why your pattern will always give the correct number of rows in Figure \( k \). Can your expression be simplified? If so, simplify it.
   e) What is the total number of tiles in each figure above?
   f) What pattern do you observe that relates the total number of tiles to the figure number? Explain in a sentence.
   g) Use this pattern to predict the total number of tiles in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.
   h) Write an algebraic expression for the total number of tiles in Figure \( k \). Explain why your pattern will always give the correct total number of tiles in Figure \( k \).
   i) Give a geometric reason why the number of tiles in Figure \( k \) is always an even number.
   j) Look at the algebraic expression you wrote in part h. Give an algebraic explanation of why this expression always gives an even number. [Hint: If your expression is not a product, use the distributive property to rewrite it as a product.]
   k) Write a summary of your thoughts and conclusions regarding Mario’s sequence of figures.
3. Mario started the discussion with Latasha wondering whether he could make a version of the diamond pattern that interested him that would use all 5000 tiles that they had in the art supplies. What do you think? Explain your thoughts. If you can use all 5000 tiles, how many rows will the design have? If a similar design cannot be made, what is the largest design that can be made with the 5000 tiles, that is, how many rows will this design have and how many tiles will be used? Explain your thoughts.

4. What is the largest design in the pattern Latasha liked that can be made with no more than 5000 tiles? How many rows does it have? Does it use all 5000 tiles? Justify your answers.

5. Let $M_1$, $M_2$, $M_3$, $M_4$, and so forth represent the sequence of numbers that give the total number of tiles in Mario’s sequence of figures.

Let $L_1$, $L_2$, $L_3$, $L_4$, and so forth represent the sequence of numbers that give the total number of tiles in Latasha’s sequence of figures.

Write an equation that expresses each of the following:

a) the relationship between $L_1$ and $M_1$   b) the relationship between $L_2$ and $M_2$

c) the relationship between $L_3$ and $M_3$   d) the relationship between $L_4$ and $M_4$

e) the general relationship between $L_k$ and $M_k$, where $k$ can represent any positive integer.

**Just Jogging Learning Task** (Rational Expressions)
For distances of 12 miles or less, a certain jogger can maintain an average speed of 6 miles per hour while running on level ground.

1. If this jogger runs around a level track at an average speed of 6 mph, how long in hours will the jogger take to run each of the following distances? [Express your answers as fractions of an hour in simplest form, the decimal equivalent of that fraction, and the equivalent number of minutes.]
   - (a) 3 miles
   - (b) 9 miles
   - (c) 1 mile
   - (d) \( \frac{1}{2} \) mile
   - (e) \( \frac{1}{10} \) mile

2. Analyze your work in Question 1. Each answer can be found by using the number of miles, a single operation, and the number 6. What operation should be used? Write an algebraic expression for the time it takes in hours for this jogger to run \( x \) miles on level ground at an average speed of 6 miles per hour.

3. Each day this jogger warms up with stretching exercises for 15 minutes, jogs for a while, and then cools down for 15 minutes. How long would this exercise routine take, in hours, if the jogger ran for 5 miles? [Express your answer as a fraction in simplest form.]

4. Let \( T \) represent the total time in hours it takes for this workout routine when the jogger runs for \( x \) miles. Write a formula for calculating \( T \) given \( x \), where, as in Question 2, \( x \) is number of miles the jogger runs. Express the formula for \( T \) as a single algebraic fraction.

5. If the jogger skipped the warm-up and cool-down period and used this additional time to jog, how many more miles would be covered? Does this answer have any connection to the answer to question 4 above?

Suppose this same jogger decides to go to a local park and use one of the paths there for the workout routine one day each week. This path is a gently sloping one that winds its way to the top of a hill.

6. If the jogger can run at an average speed of 5.5 miles per hour up the slope and 6.5 miles per hour going down the slope, how long, in hours, will it take for the jogger to cover 2 miles by going uphill for 1 mile and then returning 1 mile back down the hill? Give an exact answer expressed as a fraction in simplest terms and then give a decimal approximation correct to three decimal places.

7. If the jogger can run at an average speed of 5.3 miles per hour up the slope and 6.7 miles per hour going down the slope, how long, in hours, will it take for the jogger to cover 2 miles by going uphill for 1 mile and then returning 1 mile back down the hill? Give an exact answer expressed as a fraction in simplest terms and then give a decimal approximation correct to three decimal places.

8. Write an algebraic expression for the total time, in hours, that it takes the jogger to cover 2 miles by going uphill for 1 mile and then returning 1 mile back down the hill if the jogger
runs uphill at an average speed that is \( c \) miles per hour slower than the level-ground speed of 6 miles per hour and runs downhill at an average speed that is \( c \) miles per hour faster than the level-ground speed of 6 miles per hour. Simplify your answer to a single algebraic fraction. Verify that your expression gives the correct answers for Questions 6 and 7.

9. The average speed in miles per hour is defined to be the distance in miles divided by the time in hours spent covering the distance.
   (a) What is the jogger’s average speed for a two mile trip on level ground?
   (b) What is the jogger’s average speed for the two mile trip in question 6?
   (c) What is the jogger’s average speed for the two mile trip in question 7?
   (d) Write an expression for the jogger’s average speed over the two-mile trip (one mile up and one mile down) when the average speed uphill is \( c \) miles per hour slower than the level-ground speed of 6 miles per hour and the average speed downhill at an average speed that is \( c \) miles per hour faster than the level-ground speed of 6 miles per hour. Express your answer as a simplified algebraic fraction.
   (e) Use the expression in part (d) to recalculate your answers for parts (b) and (c)? What value of \( c \) should you use in each part?

10. For what value of \( c \) would the jogger’s average speed for the two-mile trip (one mile up and one mile down) be 4.5 miles per hour? For this value of \( c \), what would be the jogger’s average rate uphill and downhill?

\textbf{Spinner Learning Task 1} (Equally Like Outcomes)
Using only a compass and straight edge, construct a circle divided into 8 equal sectors.

Label each sector with a dollar amount in increments of $100, beginning with $100. Label the sectors in any order you choose.

Now, place the clear plastic spinner over your circle to create your own personalized spinner.

Do you think that your homemade spinner is as fair as a store bought spinner? How can you tell? If your spinner is fair and you made a dotplot for a specific number of spins, what do you think the dotplot should look like?

Spin your spinner at least 50 times and record the outcomes on your paper. Make a dotplot of your results.

Look at your dotplot. Is the shape consistent with the shape of a dotplot that would have been produced from spinning a fair spinner? Justify your answer.

Based on the shape of your dotplot, do you think 50 spins are enough to determine whether or not your spinner is fair? Explain your reasoning.

Use appropriate technology like the TI-83 Plus Probability Simulation or the National Library of Virtual Manipulatives at nlvm.usu.edu/en/nav/vlibrary.html to generate a dotplot for an increasingly greater number of spins on your spinner. At what number of spins is the shape of your dotplot consistent with the shape of a dot plot produced using a fair spinner? Why?

Work under the assumption that your spinner is fair. Answer the following questions for your spinner and explain your reasoning for each.

1) What is the probability of obtaining $800 on the first spin?
2) What is the probability of obtaining $400 on the first spin?
3) Is it just as likely to land on $100 as it is on $800?
4) What is the probability of obtaining at least $500 on the first spin?
5) What is the probability of obtaining less than $200 on the first spin?
6) What is the probability of obtaining at most $500 on the first spin?
7) If you spin the spinner twice, what is the probability that you will have a sum of $200?
8) If you spin the spinner twice, what is the probability that you will have a sum of at most $400?
9) If you spin the spinner twice, what is the probability that you will have a sum of at least $1500?
10) If you spin the spinner twice, what is the probability that you will have a sum of at least 300?

11) Given that you landed on $100 on the first spin, what is the probability that the sum of your two spins will be $200?

12) Given that you landed on $800 on the first spin, what is the probability that the sum of your two spins will be at least $1000?

13) Write a summary of your thoughts and conclusions regarding spinner 1.

Spinner Learning Task 2 (Not Equally Likely Outcomes)
Using a compass, protractor and straight edge, draw a circle with 8 sectors. Your sectors should meet the following requirements:

- You should have two sets of four congruent sectors.
- The sectors in one set should be twice as large as the sectors in the other.

How can you be sure that your circle meets these requirements? Explain your thinking.

Label each sector with a dollar amount in increments of $100 beginning with $100 and ending with $800. Place $200, $300, $400 and $800 in the larger sectors.

Now, place your clear plastic spinner over your circle to create your own personalized spinner.

Explain how this spinner is different from spinner 1.

Do you think that your homemade spinner is fair? How can you tell? Is the interpretation of fair the same for spinner 1 and spinner 2? What should a dotplot for spinner 2 look like? How is it different from the dotplot for spinner 1?

Spin your spinner at least 50 times and record the outcomes on your paper. Make a dotplot of your results. Compare your dotplot to what you predicted. Discuss any differences.

Use appropriate technology like the TI-83 Plus Probability Simulation or the National Library of Virtual Manipulatives at nlvm.usu.edu/en/nav/vlibrary.html to generate a dotplot for an increasingly greater number of spins on your spinner. At what number of spins is the shape of your dotplot consistent with the shape of a dot plot produced using a spinner designed according to the specifications given above? Why?

Based on this spinner, answer the following questions and justify your thinking.

1) What is the probability of obtaining $800 on the first spin?
2) What is the probability of obtaining $500 on the first spin?
3) Is it just as likely to land on $100 as it is on $800?
4) What is the probability of obtaining at least $500 on the first spin?
5) What is the probability of obtaining less than $200 on the first spin?
6) What is the probability of obtaining at most $500 on the first spin?
7) If you spin the spinner twice, what is the probability that you will have a sum of $200?
8) If you spin the spinner twice, what is the probability that you will have a sum of at most $400?
9) If you spin the spinner twice, what is the probability that you will have a sum of at least $1500?
10) Given that you landed on $100 on the first spin, what is the probability that the sum of your two spins will be $200?

11) Given that you landed on $800 on the first spin, what is the probability that the sum of your two spins will be at least $1500?

12) Write a summary of your thoughts and conclusions regarding spinner 2.
AFTER EXTENSIVE PLANNING, I PRESENTED what should have been a masterpiece lesson. I worked several examples on the overhead projector, answered every student's question in great detail, and explained the concept so clearly that surely my students understood. The next day, however, it became obvious that the students were totally confused. In my early years of teaching, this situation happened all too often. Even though observations by my principal clearly pointed out that I was very good at explaining mathematics to my students, knew my subject matter well, and really seemed to be a dedicated and caring teacher, something was wrong. My students were capable of learning much more than they displayed.

Implementing Change over time
THE LOW LEVELS OF ACHIEVEMENT - of many students caused me to question - how I was teaching, and my search for a - better approach began. Making a commitment to change 10 percent of my if teaching each year, I began to collect and use materials and ideas gathered from supplements, workshops, professional journals, and university classes. Each year, my goal was simply to teach a single topic in a better way than I had the year before.

Before long, I noticed that the familiar teacher-centered, direct-instruction model often did not fit well with the more in-depth problems and tasks that I was using. The information that I had gathered also suggested teaching in nontraditional ways. It was not enough to teach better mathematics; I also had to teach mathematics better. Making changes in instruction proved difficult because I had to learn to teach in ways that I had never observed or experienced, challenging many of the old teaching paradigms. As I moved from traditional methods of instruction to a more student-centered, problem-based approach, many of my students enjoyed my classes more. They really seemed to like working together, discussing and sharing their ideas and solutions to the interesting, often contextual, problems that I posed. The small changes that I implemented each year began to show results. In five years, I had almost completely changed both what and how I was teaching.

The Fundamental Flaw
AT SOME POINT DURING THIS METAMORPHOSIS, I concluded that a fundamental flaw existed in my teaching methods. When I was in front of the class demonstrating and explaining, I was learning a great deal, but many of my students were not! Eventually, I concluded that if my students were to ever really learn mathematics, they would have to do the explaining, and I, the listening. My definition of a good teacher has since changed from "one who explains things so well that students understand" to "one who gets students to explain things so well that they can be understood."

Getting middle school students to explain their thinking and become actively involved in classroom discussions can be a challenge. By nature, these
students are self-conscious and insecure. This insecurity and the effects of negative peer pressure tend to discourage involvement. To get beyond these and other roadblocks, I have learned to ask the best possible questions and to apply strategies that require all students to participate. Adopting the goals and implementing the strategies and questioning techniques that follow have helped me develop and improve my questioning skills. At the same time, these goals and strategies help me create a classroom atmosphere in which students are actively engaged in learning mathematics and feel comfortable in sharing and discussing ideas, asking questions, and taking risks.

Questioning Strategies That Work for Me

ALTHOUGH GOOD TEACHERS PLAN DETAILED lessons that focus on the mathematical content, few take the time to plan to use specific questioning techniques on a regular basis. Improving questioning skills is difficult and takes time, practice, and planning. Strategies that work once will work again and again. Making a list of good ideas and strategies that work, revisiting the list regularly, and planning to practice selected techniques in daily lessons will make a difference.

Create a plan.
The following is a list of reminders that I have accumulated from the many outstanding teachers with whom I have worked over several years. I revisit this list often. None of these ideas is new, and I can claim none, except the first one, as my own. Although implementing any single suggestion from this list may not result in major change, used together, these suggestions can help transform a classroom. Attempting to change too much too fast may result in frustration and failure. Changing a little at a time by selecting, practicing, and refining one or two strategies or skills before moving on to others can result in continual, incremental growth. Implementing one or two techniques at a time also makes it easier for students to accept and adjust to the new expectations and standards being established.

1. Never say anything a kid can say! This one goal keeps me focused. Although I do not think that I have ever met this goal completely in anyone day or even in a given class period, it has forced me to develop and improve my questioning skills. It also sends a message to students that their participation is essential. Every time I am tempted to tell students something, I try to ask a question instead.

2. Ask good questions. Good questions require more than recalling a fact or reproducing a skill. By asking good questions, I encourage students to think about, and reflect on, the mathematics they are learning. A student should be able to learn from answering my question, and I should be able to learn something about what the student knows or does not know from her or his response. Quite simply, I ask good questions to get students to think and to inform me about what they know. The best questions are open ended, those for which more than one way to solve the problem or more than one acceptable response may be possible.

3. Use more process questions than product questions. Product questions—those that require short answers or a yes or no response or those that rely almost completely on memory—provide little information about what a student knows. To find out what a student understands, I ask process questions that require the student to reflect, analyze, and explain his or her thinking and reasoning. Process questions require students to think at much higher levels.

4. Replace lectures with sets of questions. When tempted to present information in the form of a lecture, I remind myself of this definition of a lecture: "The transfer of information from the notes of the lecturer to the notes of the student without passing through the minds of either." If I am still tempted, I ask myself the humbling question "What percent of my students will actually be listening to me?"

5. Be patient. Wait time is very important. Although some students always seem to have their hands raised immediately, most need more time to process their thoughts. If I always call on one of the first students who volunteers, I am cheating those who need more time to think about, and process a response to, my question. Even very capable students can begin to doubt their abilities, and many eventually stop thinking about my questions altogether. Increasing wait time to five seconds or longer can result in more and better responses.

Good discussions take time; at first, I was uncomfortable in taking so much time to discuss a single question or problem. The urge to simply tell my students and move on for the sake of expedience was considerable. Eventually, I began to see the value in what I now refer to as a "less is more" philosophy. I now believe that all students learn more when I pose a high-quality problem and give them the necessary time to investigate, process their thoughts, and reflect on and defend their findings.
Share with students reasons for asking questions. Students should understand that all their statements are valuable to me, even if they are incorrect or show misconceptions. I explain that I ask them questions because I am continuously evaluating what the class knows or does not know. Their comments help me make decisions and plan the next activities.

Teach for success. If students are to value my questions and be involved in discussions, I cannot use questions to embarrass or punish. Such questions accomplish little and can make it more difficult to create an atmosphere in which students feel comfortable sharing ideas and taking risks. If a student is struggling to respond, I move on to another student quickly. As I listen to student conversations and observe their work, I also identify those who have good ideas or comments to share. Asking a shy, quiet student a question when I know that he or she has a good response is a great strategy for building confidence and self-esteem. Frequently, I alert the student ahead of time: "That's a great idea. I'd really like you to share that with the class in a few minutes."

Be nonjudgmental about a response or comment. This goal is indispensable in encouraging discourse. Imagine being in a classroom where the teacher makes this comment: "WOW! Brittni, that was a terrific, insightful response! Who's next?" Not many middle school students have the confidence to follow a response that has been praised so highly by a teacher. If a student's response reveals a misconception and the teacher replies in a negative way, the student may be discouraged from volunteering again. Instead, encourage more discussion and move on to the next comment. Often, students disagree with one another, discover their own errors, and correct their thinking. Allowing students to disagree with classmates is a far more positive way to deal with misconceptions than announcing to the class that an answer is incorrect. If several students remain confused, I might say, "I'm hearing that we do not agree on this issue. Your comments and ideas have given me an idea for an activity that will help you clarify your thinking." I then plan to revisit the concept with another activity as soon as possible.

Try not to repeat students' answers. If students are to listen to one another and value one another's input, I cannot repeat or try to improve on what they say. If students realize that I will repeat or clarify what another student says, they no longer have a reason to listen. I must be patient and let students clarify their own thinking and encourage them to speak to their classmates, not just to me.

All students can speak louder - I have heard them in the halls! Yet I must be careful not to embarrass someone with a quiet voice. Because students know that I never accept just one response, they think nothing of my asking another student to paraphrase the soft-spoken comments of a classmate.

Is this the right answer? Students frequently ask this question. My usual response to this question might be that "I'm not sure. Can you explain your thinking to me?" As soon as I tell a student that the answer is correct, thinking stops. If students explain their thinking clearly, I can ask a "What if?" question to encourage them to extend their thinking.

Participation is not optional! I remind my students of this expectation regularly. Whether working in small groups or discussing a problem with the whole class, each student is expected to contribute his or her fair share. Because reminding students of this expectation is not enough, I also regularly apply several of the following techniques:

1. Use the think-pair-share strategy. Whole-group discussions are usually improved by using this technique. When I pose a new problem; present a new project, task, or activity; or simply ask a question, all students must think and work independently first. In the past, letting students begin working together on a task allowed a few students to sit back while others took over. Requiring students to work alone first reduces this problem by placing the responsibility for learning on each student. This independent work time may vary from a few minutes to the entire class period, depending on the task. After students have had adequate time to work independently, they are paired with partners or join small groups. In these groups, each student is required to report his or her findings or summarize his or her solution process. When teams have had the chance to share their thoughts in small groups, we come together as a class to share our findings. I do not call for volunteers but simply ask one student to report on a significant point discussed in the group. I might say, "Tanya, will you share with the class one important discovery you made?" or "James, please summarize for us what Adam shared with you." Students generally feel much more confident in stating ideas when the responsibility for the response is being shared with a partner or group. Using the think-pair-share strategy helps me send the message that participation is not optional.

A modified version of this strategy also works in whole-group discussions. If I do not get the responses that I expect, either in quantity or quality, I give students a chance to discuss the question in small groups. On the basis of the difficulty of the question, they may have as little as fifteen seconds or as long as
LIKE MOST TEACHERS, I ENTERED THE TEACHING profession because I care about children. It is only natural for me to want them to be successful, but by merely telling them answers, doing things for them, or showing them shortcuts, I relieve students of their responsibilities and cheat them of the opportunity to make sense of the mathematics that they are learning. To help students engage in real learning, I must ask good questions, allow students to struggle, and place the responsibility for learning directly on their shoulders. I am convinced that children learn in more ways than I know how to teach. By listening to them, I not only give them the opportunity to develop deep understanding but also am able to develop true insights into what they know and how they think.

Making extensive changes in curriculum and instruction is a challenging process. Much can be learned about how children think and learn, from recent publications about learning styles, multiple intelligences, and brain research. Also, several reform curriculum projects funded by the National Science Foundation are now available from publishers. The Connected Mathematics Project, Mathematics in Context, and Math Scape, to name a few, artfully address issues of content and pedagogy.

Bibliography

7. Never carry a pencil. If I carry a pencil with me or pick up a student's pencil, I am tempted to do the work for the student. Instead, I must take time to ask thought-provoking questions that will lead to understanding.

8. Avoid answering my own questions. Answering my own questions only confuses students because it requires them to guess which questions I really want them to think about, and I want them to think about all my questions. I also avoid rhetorical questions.

9. Ask questions of the whole group. As soon as I direct a question to an individual, I suggest to the rest of the students that they are no longer required to think.

10. Limit the use of group responses. Group responses lower the level of concern and allow some students to hide and not think about my questions.

11. Do not allow students to blurt out answers. A student's blurted out answer is a signal to the rest of the class to stop thinking. Students who develop this habit must realize that they are cheating other students of the right to think about the question.

Summary
LIKE MOST TEACHERS, I ENTERED THE TEACHING profession because I care about children. It is only natural for me to want them to be successful, but by merely telling them answers, doing things for them, or showing them shortcuts, I relieve students of their responsibilities and cheat them of the opportunity to make sense of the mathematics that they are learning. To help students engage in real learning, I must ask good questions, allow students to struggle, and place the responsibility for learning directly on their shoulders. I am convinced that children learn in more ways than I know how to teach. By listening to them, I not only give them the opportunity to develop deep understanding but also am able to develop true insights into what they know and how they think.

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Bibliography
The Article: *Tips and Strategies for Co-Teaching at the Secondary Level*, Teaching Exceptional Children, Vol. 36, No. 5, pp. 52-58, could not be posted online at this time. For access please visit the Council for Exceptional Children Website at: http://www.cec.sped.org/AM/Template.cfm?Section=Home&TPLID=26&CONTENTID=4723&THISPAGE=2&TEMPLATE=/TaggedPage/TaggedPageDisplay.cfm&TPPID=1173
Please note that in order to download the article electronically you must be a member of the Council for Exceptional Children which requires a payment of annual dues.
The Equalizer

Concrete to abstract
(representations, ideas, applications, materials)

Simple to complex
(resources, research, issues, problems, skills, goals)

Basic to transformational
(information, ideas, materials, applications)

Single facets to multi-facts
(directions, problems, applications, solutions, approaches, disciplinary connections)

Smaller leaps to greater leaps
(application, insight, transfer)

More structured to more open
(solutions, decisions, approaches)

Less independence to greater independence
(planning, designing, monitoring)

Slow to faster
(pace of study, pace of thought)

Tomlinson
What should we see in a standards-based mathematics classroom?

- **Warm-up**
  - connected to the lesson
  - starts students thinking in right direction

- **Mini lesson, opening, setting the stage**
  - checks for prior knowledge
  - reviews needed skills
  - left in view for quick access during work period

- **Work period, Activity period**
  - rigorous mathematics
  - use of previously learned concepts in service of new ideas
  - collaboration and verbalization
  - process skills
  - individual accountability

- **Summary, Closing**
  - presentation and comparison of different approaches
  - students commenting on and questioning the approaches of other students
  - teacher guiding the discussion, if necessary, to solidify concepts, skills and procedures to be learned
  - clarifying of misconceptions
## Co-Teaching Models Between General and Special Education Teachers

<table>
<thead>
<tr>
<th>Assisted Teaching</th>
<th>Station Teaching</th>
<th>Parallel Teaching</th>
<th>Alternative Teaching</th>
<th>Team Teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Lead teacher models organization of the content.</td>
<td>• Lead teacher and support teacher segment the lesson content.</td>
<td>• Lead teacher and support teacher collaborate to organize the lesson content.</td>
<td>• Lead teacher and support teacher make decisions about the content and organization of the lesson.</td>
<td>• Lead teacher and support teacher make decisions about the content and organization of the lesson.</td>
</tr>
<tr>
<td>• Lead teacher identifies skills and strategies needed for groups and individual students to complete the task of the lesson.</td>
<td>• Lead teacher and support teacher divide the number of stations they are responsible for.</td>
<td>• Lead teacher and support teacher identify strategies needed for groups and individual students.</td>
<td>• Lead teacher and support teacher determine the appropriate structures for alternative remedial or enrichment lessons that would promote student learning.</td>
<td>• Lead teacher and support teacher teach simultaneously to whole class.</td>
</tr>
<tr>
<td>• Support teacher assists.</td>
<td>• Both teachers plan and organize their station activities with attention to possible group differences.</td>
<td>• Lead teacher and support teacher divide the students into two groups.</td>
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### Design
- Lead teacher conducts formal teaching.
- Support teacher teaches components of lessons with small groups of individuals.
- Support teacher provides content support to lead teacher’s lesson.

### Communication
- Lead teacher and support teacher segment learning to small groups or individual at the stations they design.

### Parallel Teaching
- Lead teacher and support teacher independently deliver the lesson plan to each of the groups.
- Lead teacher and support teacher facilitate learning in their group.

### Alternative Teaching
- Lead teacher conducts formal teaching.
- Support teacher implements supplemental activities for the whole group, small groups or individuals before or after the formal lesson.

### Team Teaching
- Both lead and support teacher conducts formal teaching.
## Co-Teaching Models Between General and Special Education Teachers

<table>
<thead>
<tr>
<th>Assisted Teaching</th>
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<tbody>
<tr>
<td>- Lead teacher uses pre-assessment to determine students' need for support.</td>
</tr>
<tr>
<td>- Support teacher assesses students' skills and facilitates self-regulation during the lesson.</td>
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<tr>
<td>- Students use self-assessment as they request assistance during or after a formal lesson.</td>
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<table>
<thead>
<tr>
<th>Station Teaching</th>
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<tbody>
<tr>
<td>- Lead teacher and support teacher use pre-assessment to determine how students are selected for stations (e.g., skills, interests, random, etc).</td>
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<tr>
<td>- Given the organizational structure and tasks of each station, assessment done by students can be used during the lesson.</td>
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<table>
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<tr>
<th>Parallel Teaching</th>
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<tbody>
<tr>
<td>- Lead teacher and support teacher monitor their own groups of students.</td>
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<tr>
<td>- Lead teacher and support teacher use post lesson reflection to share their expectations using the same lesson plan with different groups of students.</td>
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<table>
<thead>
<tr>
<th>Alternative Teaching</th>
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<tbody>
<tr>
<td>- Lead teacher and support teacher pre-assess the students to plan for alternative lessons.</td>
</tr>
<tr>
<td>- Lead teacher and support teacher assess the students during the formal lesson to identify students who would benefit from the alternative lessons.</td>
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<tr>
<td>- Student self-assessment and/or peer-assessment encourages students to articulate their need for alternative forms of instruction.</td>
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<tr>
<th>Team Teaching</th>
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<td>- Lead teacher and support teacher pre-assess the students.</td>
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### Benefits

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<tr>
<th><strong>Co-Teaching</strong></th>
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<tr>
<td><strong>Monitoring</strong></td>
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<tr>
<td>Facilitates small group learning and is responsive to individual needs. The notions of &quot;mimilesson&quot;, &quot;mastery learning&quot;, &quot;accelerated learning&quot;, and other ideas that teach to mastery levels can be readily addressed in this mode. Parallel teaching is very helpful whenever we want to increase the likelihood of participation, publication, and sharing. Also, it allows us to work intensively with a small group of students. Allows us to use alternative methods to re-teach or extend the lesson up or down. This model reminds us that we may need more visual, auditory, tactile, kinesthetic support to successfully communicate certain skills, concepts and ideas. Team teaching is very powerful when the entire class is participating in a particular inquiry project like a thematic unit.</td>
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<tr>
<td><strong>Co-Teaching</strong></td>
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<td><strong>Benefits</strong></td>
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63
Georgia Student Achievement Pyramid of Interventions

Collaboratively Developed by the Georgia Department of Education
Departments of Curriculum & Instruction and Teacher & Student Support

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Student Achievement Pyramid of Interventions

Wanda Creel, Sally Kristel, John O’Connor, Lynne Williams

The Georgia Department of Education is unveiling a conceptual framework that will enable all students in Georgia to continue to make great gains in school. The Student Achievement Pyramid of Interventions is the result of teamwork and collaboration throughout the Georgia Department of Education. The catalyst for the development and implementation of Georgia’s Student Achievement Pyramid of Interventions was the need for Georgia’s educators to have a common focus and a common language regarding instructional practices and interventions. The Student Achievement Pyramid of Interventions is a graphic organizer that illustrates layers of instructional efforts that can be provided to students according to their individual needs. Additionally, the Student Achievement Pyramid of Intervention can serve as a framework for discussion among collaborative professional learning communities that are willing to explore and engage in all avenues available to assist students in their learning process.

While the Student Achievement Pyramid of Intervention may sound like “educational jargon,” it provides a framework to align practices with the mission of learning for everyone. Richard DuFour says that pyramids of intervention prod us to ask the following questions: “Are our kids learning? How do we know that they are learning? And, most importantly, what are we prepared to do when they do not learn?” DuFour asserts that the final question is the distinguishing characteristic of a professional learning community.

In a professional learning community, DuFour states, there is a commitment to help students learn, but the commitment goes much deeper than in schools without professional learning communities. “In the professional learning community, we say that learning is so important that we are going to do whatever it takes to help you learn, and we are not going to let up on you until you do learn.”

Research consistently points out that student growth is enhanced when evaluation results are used to guide continued instruction. This concept of monitoring students’ progress, or “progress monitoring,” enables educators to determine if students are increasing their skills as expected, or if they need additional instructional interventions to enable them to maximize academic success. The Student Achievement Pyramid of Interventions represents the process of continually implementing “progress monitoring” and then providing layers of more and more intensive interventions so that students can be successful and progress in their learning. This proactive approach does not wait until students have large gaps in their learning that are almost too great to overcome. Neither does it allow high-achieving students to languish in a curriculum that is not challenging to them. This approach focuses on determining when students are struggling and providing strategic interventions to help them shore up their areas of need; it also documents students’ strength and provides additional challenge in a variety of ways. Georgia’s Student Achievement Pyramid of Interventions begins with standards-based classrooms serving as the foundation for teaching and learning.

Pages 3-4 explain the tiered levels of Georgia’s Student Achievement Pyramid of Intervention
STUDENT ACHIEVEMENT PYRAMID OF INTERVENTIONS

Tier 1: Standards Based Classroom Learning describes effective instruction that should be happening in all classrooms for all students. As Georgia moves towards phasing in the implementation of the Georgia Performance Standards (GPS) it is recognized that the curriculum standards are the foundation for the learning that occurs in each classroom. This type of instruction/learning focuses on the GPS and includes evidenced based instruction that is differentiated according to students' various needs. Teachers utilize progress monitoring results to guide and adjust instruction. Tier 1 is not limited to instruction in the academic content areas, but also includes all developmental domains such as behavioral and social development. This tier represents effective, strategic, and expert instruction that is ideally available in all classrooms. Through standards-based learning and ongoing formative assessments we can answer DuFour's questions of "are kids learning; and how are they learning?"

Tier 2: Needs Based Instruction/Learning: Standard Intervention Protocols: Tier Two becomes the answer to the question "what are we prepared to do when they do not learn?" Tier 2 describes pre-planned interventions that should be in place for students who are not being sufficiently successful or adequately challenged with Tier 1 interventions alone. In many schools in Georgia, students who need additional interventions in the general classroom have been referred to the Student Support Team and possibly evaluated for special education services. The new conceptual framework illustrates the potential for having interventions for students before their gap becomes so large that specialized instruction is needed. Tier 2 interventions are not a substitution for Tier 1 interventions, but are layered in addition to the Tier 1 instruction that is provided. Tier 2 interventions are not solely reliant on the expertise and diligence of individual teachers across the school. They should include pre-planned interventions developed and supported at the school level, thereby becoming "standard intervention protocols" that are proactively in place for students who need them. Working collaboratively, teachers and instructional leaders should determine concepts and content areas that have traditionally proven difficult for students in their school. Then, they should develop interventions that are available when specific students show weaknesses in those areas. For 1st and 2nd-grade students who struggle with learning to read, for example, Tier 2 interventions may include structured, diligent, pre-planned tutoring interventions for those specified students. Similarly, schools should determine concepts and content areas that are likely to have been mastered by highly able students and, through strategies such as pretesting and curriculum compacting, be prepared to provide modified curriculum. All students who need a Tier 2 intervention (in addition to their Tier 1 instruction) should be identified through the progress monitoring evaluation data.
Tier 2 interventions can be used at all school levels. Virtually every high school has students who become disenfranchised and unsuccessful and therefore become high risk for dropping out of school. High schools, possibly in collaboration with local middle schools, can anticipate this and identify those students very early in their high school careers who are high risk for this type of difficulty. They can then build systematic mentoring programs that encourage students to become active and engaged in high school activities. In addition, specific academic interventions can be established for students who are missing core academic skills (e.g., strong reading skills) that will increase the probability that high risk students will have the necessary skills to be successful. To maintain motivation and improve academic achievement, high schools should use a variety of strategies to encourage more students to engage in rigorous coursework, e.g., vertical teaming that leads to AP courses.

Tier 2 interventions should not be endless for individual students who are struggling. Schools must ensure that specific students are not labeled as being “Tier 2 students” and thereby create lower expectations or tracking for those students. Tier 2 interventions are proactive and maintain high expectations for all students.

**Tier 3: Student Support Team Driven Instruction/Learning** provides an additional layer of analysis and interventions. The Student Support Team (and other small group teams such as the Gifted Eligibility Team) meet to discuss students who are still not provided the instructional experiences to meet their needs. During this process, the diagnostic team analyzes the specific needs of the individual student. In Tier 2, schools establish standard intervention protocols that are available across the school. Tier 3 becomes much more individualized as the student’s teachers, other personnel, and parents systematically determine the issues that need to be addressed for the student. Instructional interventions are then strategically put in place for the student and progress monitoring processes, including sensitive instruments that may be formal or informal in nature, are implemented frequently to determine if the student is responding to the interventions. Effective Tier 3 activities are exemplified by systematic activities to determine a student’s needs, implementation of scientifically-based interventions that are strategically incorporated with fidelity to meet the student’s individual needs, and frequent progress monitoring to inform continued instruction.

**Tier 4: Specially Designed Instruction/Learning** is developed specifically for students who meet the respective eligibility criteria for special program placement. With three effective tiers in place prior to specialized services, more struggling students will be successful and will not require this degree of interventions. Tier 4 will provide instruction that is targeted and specialized to meet students’ needs. Tier 4 instruction would include formal Gifted Education services for students who qualify, but it may also include interventions suggested by the Gifted Eligibility Team for regular classroom curriculum modification for any student with advanced learning needs. It may include special education and related services for eligible students, provided in the general education classroom, or in some cases, in a resource room. Tier 4 does not represent a location for services, but indicates a layer of interventions that may be provided in the general education class or in a separate setting. Tier 4 is not a substitute for Tier 2, but is layered upon Tier 2 interventions.
Multiple Representations

Instructional programs from prekindergarten through grade 12 should enable all students to:

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

If mathematics is the "science of patterns" (Steen 1988), representations are the means by which those patterns are recorded and analyzed. As students become mathematically sophisticated, they develop an increasingly large repertoire of mathematical representations and the knowledge of how to use them productively. This knowledge includes choosing specific representations in order to gain particular insights or achieve particular ends.

In grades 9–12, students' knowledge and use of representations should expand in scope and complexity. As they study new content, for example, students will encounter many new representations for mathematical concepts. They will need to be able to convert flexibly among these representations. Much of the power of mathematics comes from being able to view and operate on objects from different perspectives.

National Council of Teachers of Mathematics

*Principles and Standards for School Mathematics* (Representation Standards for Grades 9-12)
## Middle School Mathematics Vertical Alignment Chart

<table>
<thead>
<tr>
<th></th>
<th>6th Grade</th>
<th>7th Grade</th>
<th>8th Grade</th>
</tr>
</thead>
</table>
| **Numbers and Operations** | • Factors and multiples  
• Fundamental Theorem of Arithmetic  
• GCF and LCM  
• Compute with fractions and mixed numbers (unlike denominators)  
• Equivalent fractions, decimals, and percents | • Absolute value  
• Compare & order rational numbers  
• Compute & solve problems with positive and negative rational numbers | • Square roots of perfect squares  
• Rational vs Irrational numbers  
• Simplify expressions with integer exponents  
• Scientific Notation |
| **Measurement**      | • Convert units using proportions  
• Volume of right rectangular prisms, right circular cylinders, pyramids and cones  
• Surface area of right rectangular prisms, right circular cylinders | • Basic constructions  
• Transformations  
• Properties of similarity  
• 3-D figures formed by translations & rotations in space  
• Cross sections of cones, cylinders, pyramids and prisms | • Properties of parallel and perpendicular lines  
• Meaning of congruence  
• Pythagorean Theorem |
| **Geometry**         | • Line & rotational symmetry  
• Ratio, proportion and scale factor with similar plane figures  
• Scale drawings  
• Compare/contrast right prisms/pyramids and cylinders/cones  
• Views of solid figures  
• Nets (prisms, cylinders, pyramids, and cones) | • Algebraic expressions  
• Linear equations in one variable  
• Relationships between two variables | • Represent, analyze, and solve problems  
• Inequalities in one variable  
• Graphs of linear equations and inequalities  
• Systems of linear equations and inequalities |
| **Algebra**          | • Ratio for quantitative relationship  
• Write & solve proportions  
• Write & solve simple one-step equations | • Question, Collect Data, Make Graphs, Interpret results | • Set theory  
• Tree Diagrams/ Counting Principles  
• Basic laws of probability  
• Organize, interpret, make inferences from data |
| **Data Analysis and Probability** | • Question, Collect Data, Make Graphs  
• Experimental/Theoretical Probability  
• Predictions from investigations | • Question, Collect Data, Make Graphs, Interpret results | • Problem Solving, Arguments, Communicate, Connections, Multiple Representations |
| **Process Skills**   | Problem Solving, Arguments, Communicate, Connections, Multiple Representations | Problem Solving, Arguments, Communicate, Connections, Multiple Representations | Problem Solving, Arguments, Communicate, Connections, Multiple Representations |
## High School Mathematics 1 Vertical Alignment Chart

<table>
<thead>
<tr>
<th></th>
<th>MATH 1</th>
<th>MATH 2</th>
<th>MATH 3</th>
<th>MATH 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NUMBER &amp; OPERATIONS</strong></td>
<td>• Complex numbers</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td><strong>GEOMETRY</strong></td>
<td>• Distance between 2 points</td>
<td>• Special right triangles</td>
<td>• Investigate relationships between lines and circles</td>
<td>• Circle</td>
</tr>
<tr>
<td></td>
<td>• Distance between a point and a line</td>
<td>• Right triangle trigonometry</td>
<td>• Ellipse</td>
<td>• Hyperbola</td>
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<tr>
<td></td>
<td>• Midpoint</td>
<td>• Circles and properties</td>
<td>• Parabola</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Inductive, deductive reasoning</td>
<td>• Length of arc</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Converse, inverse, contraposition</td>
<td>• Area of a sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Sum of interior, exterior angles</td>
<td>• Surface area and volume of sphere</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Triangle inequalities</td>
<td>• Relationships of similar solids</td>
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<tr>
<td></td>
<td>• SSS, SAS, ASA, AAS, HL</td>
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<tr>
<td></td>
<td>• Use and prove properties of special quadrilaterals</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>• Incenter, orthocenter, circumcenter, centroid</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
### MATH 1
- Function notation
- \( F(x) = x^n \) (\( n=1,2,3 \), \( \sqrt{x} \cdot |x| \), and \( 1/x \))
- Characteristics of these functions
- Sequences as functions
- Compare rates of change among functions
- Even, odd, neither
- Simplify expressions involving and perform operations with square roots
- Operations on polynomials
- Binomial theorem
- Factoring of 2nd degree polynomials & cubes
- Solve quadratic equations with \( a=1 \)
- Solve equations using radicals
- Solve simple rational equations (\( a=1 \) only)

### MATH 2
- Step & piecewise functions
- Characteristics of their graphs
- Solve absolute value equations and inequalities
- Exponential functions
- Solve exponential equations and inequalities
- Geometric sequences as exponential function
- Quadratic function (\( y = ax^2 + bx + c \) and its graph)
- Quadratic inequalities
- Inverses

### MATH 3
- Polynomials of degree \( > 2 \)
- Classify polynomial functions as even, odd, or neither
- Characteristics of poly. functions
- Logarithmic functions
- Solve exponential, logarithmic and polynomial equations and inequalities
- Perform operations with, find inverses of, and examine properties of matrices
- Use matrices of represent and solve problems
- Linear programming
- Vertex-edge graphs

### MATH 4
- Rational functions
- Solve rational equations and inequalities
- Unit circle trigonometric functions
- Graph of 6 trigonometric functions
- Build functions using sum, difference, product, quotient, and composition of functions
- Trigonometric identities
- Solve trigonometric equations and inequalities by graphing and algebraic manipulation
- Law of Sines
- Law of Cosines
- Area of triangle (trig) formula
- Inverse trigonometric functions (sine, cosines, and tangent only)
- Sequences and series
- Summation notation
- Understand and use vectors

### DATA ANALYSIS AND PROBABILITY
- Principles of counting
- Simple permutations & combinations
- Mutually exclusive, dependent, and conditional events
- Expected values
- Compare summary statistics
- Understand random sample
- Mean absolute deviation

### PROCESS STANDARDS
- Problem Solving, Arguments, Connections, Multiple Representations, Communication
- Problem Solving, Arguments, Connections, Multiple Representations, Communication
- Problem Solving, Arguments, Connections, Multiple Representations, Communication
- Problem Solving, Arguments, Connections, Multiple Representations, Communication

### Central limit theorem
- Confidence interval
- Margin of error
Additional Resources

Web sites for Learning Style and Multiple Intelligences surveys:

http://lookingahead.heinle.com/filing/l-styles.htm
Perceptual Learning Style Preference Questionnaire – hard copy survey for students

http://www.engr.ncsu.edu/learningstyles/ilsweb.html
online Learning Styles Questionnaire that provides immediate feedback in a variety of areas

http://www.humanmetrics.com/cgi-win/JTypes2.asp
online survey for Jungian-based profile – detailed information on types can be found at http://typelogic.com/

http://www.thirteen.org/edonline/concept2class/mi/index.html
includes lots of information on Multiple Intelligences, including links to lessons for K-12 classrooms (be sure to click through Explanation-Demonstration-Exploration-Implementation at the top)

http://www.thirteen.org/edonline/concept2class/mi/index.html
the Paragon Learning Style Inventory is also aligned with Jungian theory and a hard copy survey and explanation is available here

http://surfaquarium.com/MI/inventory.htm
this site offers a hard-copy version of the MI test

http://www.mitest.com/o2ndary.htm
this site offers an online version of the MI test for ages 13 to 18 (only 7 of Gardner’s intelligences are used)

http://www.mitest.com/o7inte~1.htm
this site offers an online version of the MI test for adults (only 7 of Gardner’s intelligences are used)

http://www.literacyworks.org/mi/assessment/findyourstrengths.html
an online assessment for multiple intelligences

http://www.chaminade.org/inspire/learnstl.htm
this site presents a chart of learning styles (visual, auditory, tactile) – answer questions to understand your own learning style better
Resources for Differentiation:

Books that offer specific strategies, as well as lessons and units that incorporate differentiated instruction:


Resources for Co-Teaching:

Video: The Power of 2; by Marilyn Friend

Video: Collaborative Planning and Collaborative Teaching; by Richard Villa

Video: How to Co-teach to Meet Diverse Student Needs; by ASCD

http://www.powerof2.org
This website offers an array of support and information about collaboration. There are links, online training, FAQs and articles.

http://wblrd.sk.ca/~bestpractice/index.html#
This website offers ideas for differentiated instructional strategies, including learning contracts, tiered instruction, and anchor activities (click on INDEPENDENT STUDY at the left)

http://www.nichcy.org
Extensive information on disabilities and disability-related issues for families, teachers, and others. The website includes fact sheets and research briefs on specific disabilities as well as special education law. Many of the materials available for downloading are available in Spanish as well as English.
The National Association of the Deaf
http://www.nad.org/
The NAD advocates for people who are deaf or hard of hearing, provides information on topics related to hearing loss, and serves individuals with these special needs.

Councils of Educators for Students with Disabilities
http://www.504idea.org/504resources.html
Wide variety of presentation materials and papers related to Section 504

@ctivTeen
http://www.disabilitycentral.com/activteen/about.htm
Managed by teenagers who have disabilities and designed especially for them. The site includes an e-zine as well as message boards and activities pages.

Internet Resources for Special Children (IRSC)
http://www.irsc.org
Provides numerous links to resources about disabilities and special education issues.

Special Education Resources on the Internet (SERI)
http://www.seriweb.com/
Devoted to individuals with disabilities. Topics to explore on the site include legal resources, parent resources, medical resources, and specific disability categories.

Federally-compiled annual reports to Congress on the implementation of IDEA
http://www.ed.gov/about/reports/annual/osep/index.html
Statistics about students with disabilities and how they receive their education services. Reports available from the past several years. The reports begin with summaries on essential issues, but also include extensive appendices with a wide variety of data tables.