Centers of Triangles Learning Task

Unit 3

Course

Mathematics I: Algebra, Geometry, Statistics

Overview

This task provides a guided discovery and investigation of the points of concurrency in triangles. Students will construct and use the following points:

- incenter
- orthocenter
- circumcenter
- centroid

In this task students will determine the location for an amusement park by finding the centers of a triangle. The centers will be new to the students but not the constructions. Make sure students remember the significance of points on the perpendicular bisector of a segment (equidistant from the endpoints of the segment) and the points on an angle bisector (equidistant from the sides of the angle).

As students work through the tasks and present their solutions to the class make sure they emphasize the name of the center found, how it was found, and its significance.

The significance of the circumcenter and incenter can be determined through measurement. If needed, encourage students to measure the distances from the triangle centers to the sides and vertices of the triangle. Students can use properties of the perpendicular bisectors and angle bisectors to justify their conjectures about the significance of the circumcenter and the incenter. Students may require help in determining the significance of the centroid (center of gravity). Students can determine one of the significant features of the centroid through measurement (the centroid is twice the distance from the vertex to the opposite side). Other than being the point of concurrency of the altitudes the orthocenter has no additional significance in this task.

Triangle Center:	*Point of Concurrency of:*	*Significance of:*
Incenter	Angle bisectors	Center of inscribed circle Equidistant from the sides of the triangle
Circumcenter	Perpendicular bisectors	Center of the circumscribing circle Equidistant from the vertices of the
Orthocenter	Altitudes	
Centroid	*Medians*	*Center of balance or gravity* *The distance from a vertex to the centroid is twice the distance from the centroid to the opposite side.*

Key Standards

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

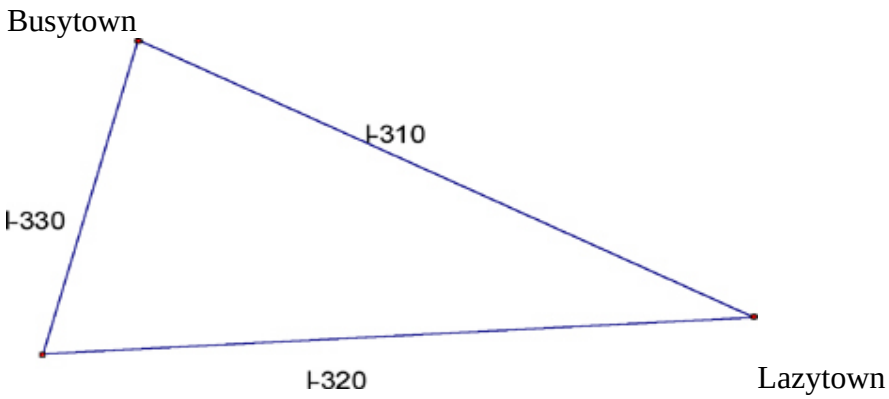
e. Find and use points of concurrency in triangles: incenter, orthocenter, circumcenter, and centroid.

Possible Materials

• Miras (or a similar reflective tool)
• patty paper
• compasses
• straightedges
• Geometer's Sketchpad or similar geometry software, if possible

Task

A developer plans to build an amusement park but wants to locate it within easy access of the three largest towns in the area as shown on the map below. The developer has to decide on the best location and is working with the ABC Construction Company to minimize costs wherever possible. No matter where the amusement park is located, roads will have to be built for access directly to the towns or to the existing highways.



Sample Questions/Solutions

1.
Just by looking at the map, choose the location that you think will be best for building the amusement park. Explain your thinking.

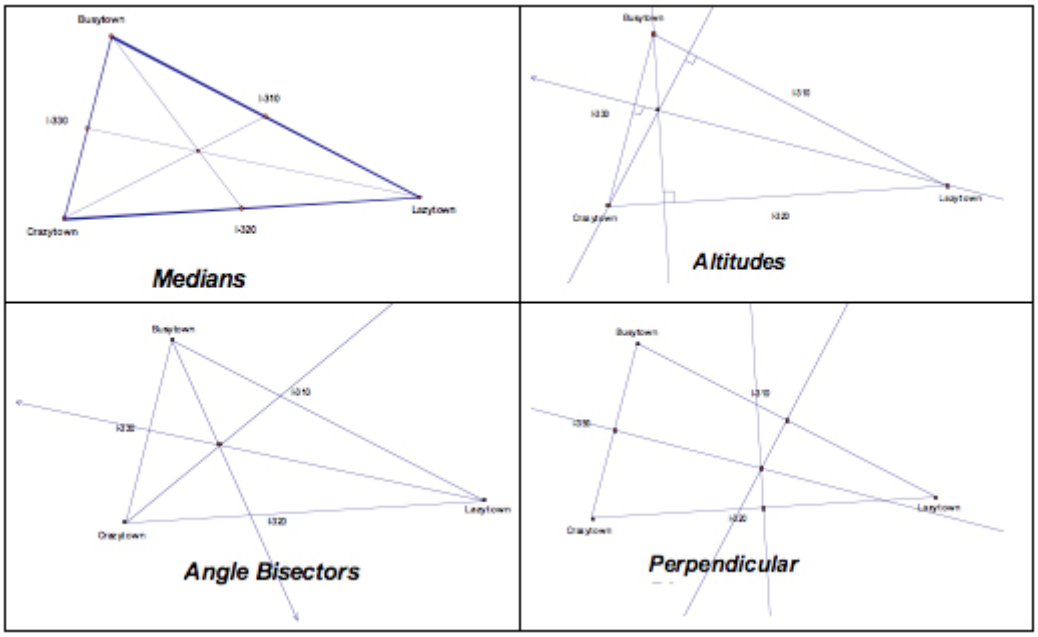
Comments:
"Just by looking" is important here. Students need to take a moment to look at the towns and make a decision.

Solution:
Answers will vary.

2.
Now you will use some mathematical concepts to help you choose a location for the tower.
In the previous lesson, you learned how to construct medians and altitudes of triangles. In 7th grade, you learned how to construct angle bisectors and perpendicular bisectors. Investigate the problem above by constructing the following:
a) all 3 medians of the triangle
b) all 3 altitudes of the triangle
c) all 3 angle bisectors of the triangle
d) all 3 perpendicular bisectors of the triangle

You have four different kinds of tools at your disposal- patty paper, MIRA, compass and straight edge, and Geometer's Sketch Pad. Use a different tool for each of your constructions.

Not drawn to scale:
The constructions, regardless of which tool is used, should result in the following. It is very important for students to realize the three lines always intersect in a single point. Because some of the tools they are using are not very precise the students will have some errors in measurement. These errors should be discussed as a group but some conclusions can still be made even with these errors.



3.
Choose a location for the amusement park based on the work you did in part 2. Explain why you chose this point.

Solution:
Answers will vary but it is critical for students to have a mathematical justification for their decision. For example, they may choose the circumcenter because it is equidistant from all three cities. Or they may choose the incenter because it is equidistant from each of the roads. They could choose the centroid instead of the circumcenter because it is closer to two of the cities while not being that much further away from Lazytown.

4.
How close is the point you chose in part 3, based on mathematics, to the point you chose by observation?

Solution:
Answers will vary.

You have now discovered that each set of segments resulting from the constructions above always has a point of intersection. These four points of intersection are called the points of concurrency of a triangle.
The intersection point of the medians is called the centroid of the triangle.
The intersection point of the angle bisectors is called the incenter of the triangle.
The intersection point of the perpendicular bisectors is called the circumcenter of the triangle.
The intersection point of the altitudes is called the orthocenter of the triangle.

Comments:
Students struggle with using the terms point of concurrency and concurrent lines correctly. Make sure they understand what they mean and how to use them. To help them understand the significance of a point of concurrency, ask them to draw three lines on their paper, without looking at anyone else's paper. Then, ask whose lines are drawn in such a way that all three intersect at the same point. Have them compare their drawings and determine the different ways three lines can be related to each other.

5.
Can you give a reasonable guess as to why the specific names were given to each point of concurrency?

Comments:
Students will need to have an idea of the significance of the triangle centers in order to answer this question. If they have not already done so, they need to go back to their constructions and explore the properties of the triangle centers they found in #2. Some groups may have discovered these properties already.

Make sure the name of the center, how it was found, and its significance are emphasized as students present their solutions.

Solution:
Answers will vary.

6.
Which triangle center did you recommend for the location of the amusement park?

Solution:
The students are only being asked to name their point. They need to decide if their chosen point is the centroid, incenter, circumcenter, or orthocenter based on the definitions above.

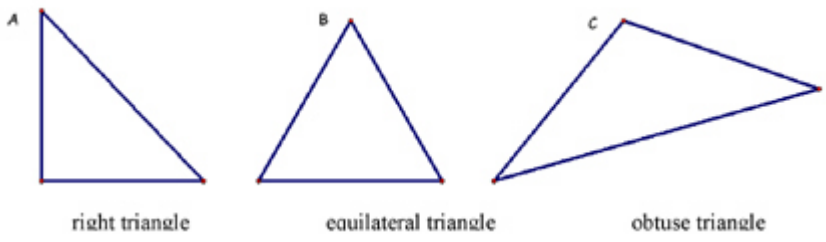
7.
The president of the company building the park is concerned about the cost of building roads from the towns to the park. What recommendation would you give him? Write a memo to the president explaining your recommendation.

Comments:
Since the president is concerned about the cost of the roads, students need to take that into account in their memo. In consideration of the cost of building the roads, some groups may want to change their earlier decision. For example, if a group chose to use the circumcenter because it was an equal distance from each of the cities they may want to choose the incenter or median to reduce the cost of building the roads. Or, students could measure the total distance from a point of concurrency to each of the cities and choose the center that gives the shortest total distance.

Solution:
Answers may vary. Mathematical justification of the answer is the most important aspect of this activity.

Sample Lesson – Day 1

Essential Question(s) :	How are the medians, altitudes, angle bisectors and perpendicular bisectors of triangles constructed?
Suggested Materials :	Miras, compasses, straightedges, patty paper, Geometer's Sketchpad
Warm Up :	Define and illustrate the following terms: median of a triangle, altitude of a triangle, angle bisector and perpendicular bisector.
Opening Mini Lesson :	Review how to use the tools provided for basic constructions, including midpoints, angle bisectors and perpendicular lines.
Task Time :	**Watch for:** Correct use of materials for constructions. Make sure that students are finding points of concurrency. **Guiding Questions:** Where are your lines/segments intersecting? How do these points of intersection relate to the vertices and sides of the triangle?
Summary / Closing :	Discuss the possible locations for each of the points of intersection of the medians, altitudes, angle bisectors and perpendicular bisectors of triangles.
Homework :	Use patty paper to construct the medians, angle bisectors, perpendicular bisectors and altitudes of the following triangles.



Sample Lesson Plan – Day 2

Essential Question(s) :	What are the points of concurrency of a triangle? How are these points used?
Warm Up :	Give each student a notecard to write a term or phrase describing last night's homework. Have students take turns posting their notecards somewhere in the room (bulletin board or whiteboard are best locations). No terms or phrases may be repeated. Briefly discuss their observations.
Opening Mini Lesson :	Review homework from day 1. Discuss possible locations for each point of concurrency.
Task Time :	**Watch for:** Did the students actually find points of concurrency? Did students locate and label the centers correctly? Are students measuring distances correctly? **Guiding Questions:** Which center is equidistant from the vertices? Which center is equidistant from the sides? What is the most important criteria to consider when selecting the point for the amusement park? Which point will reduce the costs of building roads? Which center did you select?
Summary / Closing :	Group presentations – Presentations should include the group's location for the amusement park and justification.
Homework :	After the presentations and class discussion, each student should write the requested memo.

Sample Assessment

Describe a procedure for locating the point that is the center of a circular paper disk. Use geometric definitions, properties, or principles to explain why your procedure is correct. Use the disk provided to help you formulate your procedure. You may write on it or fold it in any way that you find helpful, but it will not be collected.