A Second Challenge: Constructing a Reflection

Unit 4: Flip, Slide and Turn

Grade Level

Grade 7

Overview

In this task, students will construct a reflection of a triangle over a given line of symmetry. Also the students will create the line of symmetry using the reflected triangle.

Key Standards

M7G2. Students will demonstrate understanding of transformations.

a. Demonstrate understanding of translations, dilations, rotations, reflections and relate symmetry to appropriate transformations.

Possible Materials

- overhead projector
- colored pencils
- A Second Challenge student handout
- transparency paper
- compass

Task

A Second Challenge: Constructing a Reflection

In the drawing below, line *l* is perpendicular to line *m*. You can check this by folding the paper along line *m* and seeing that line *l* reflects upon itself. Can you construct a reflection of $\triangle$ABC using only Euclidean tools and line *m* as a line of reflection? Label the corresponding point for vertex A as A', B as B', and C as C'.

Sample Questions

Sample Question Solutions

Assessment Ideas

For this construction, students may want to work with a partner or in small groups, discussing strategies and recording the steps that they used. Up to this point, students have only been exposed to copying line segments and constructing circles, therefore students should be able to construct the reflection by copying segments. In the eighth grade, students will learn that this construction uses the theorem of Side-Side-Side congruence.

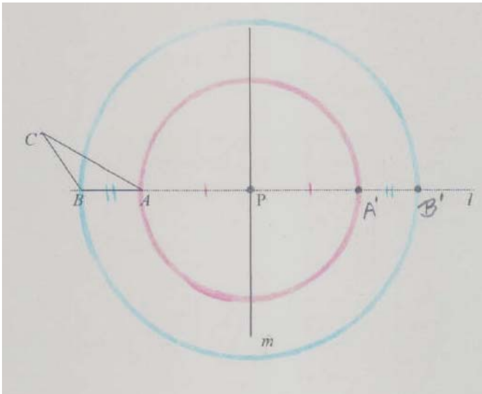
Students should develop the notion that all constructions in this task are based upon the construction of congruent circles. Since a circle is defined as the set of all points in a plane that are the same distance (called the radius) from a given point, the process of copying a segment is simply copying a radius to a new point. Students are instructed to use circles, rather than arcs, to help them justify what segments are congruent by associating them with the radii of congruent circles. It may be helpful to have students use colored pencils in their construction, using a different colored pencil for each different radius that they use. This will assist them in recognizing congruent segments.

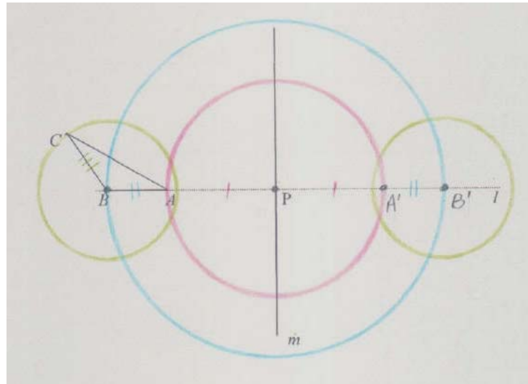
In their construction, students may choose to first find the reflection of point A by constructing a circle with radius AP, or alternatively find the reflection of point B using radius PB. After finding the point of reflection, they can copy $\overline{AB}$ *to the new point of reflection. (Instead, they could reflect both A and B and then find C by drawing a circle with radius PC and then determining C′ by constructing either A′C′ or B′C′.) The construction can be completed by constructing two remaining circles with radii BC and AC on the endpoints of the reflection of* $\overline{AB}$*.*

In the student explanations, it is critical that students recognize that the original point and its image (or the point created by the reflection) are equidistant from the line of reflection. They should also make the logical connection that this distance can be described by the radius of a circle centered at point P. The two follow up questions are designed to have students think again about the distance and the definition of congruence. Students should verify that their triangle is congruent by folding the paper along the reflection line and identifying all three corresponding sides and all three corresponding angles as congruent. During discussions teachers may want to lead the students to notice that one way to copy a triangle is to copy each side. This concept of congruence is important for later exercises.

The final question asks students to explore their construction. Teachers may want to provide copies of an accurate reflection construction for those who have challenges with fine motor skills and to ensure that they can make correct observations. Putting this on a transparency will allow the student to place the accurate construction on top of their own to help with the student's self-assessment. On the final question students should observe that all circles constructed will intersect twice with the line of reflection. Why? If those points of intersection are connected with the centers of the circles they form congruent radii and congruent triangles (using line m, the radii, and the segment contained in the line of reflection. Students can verify this by folding on the reflection line and observing congruent parts. This activity and the accompanying student observations are essential to completing the next activity.

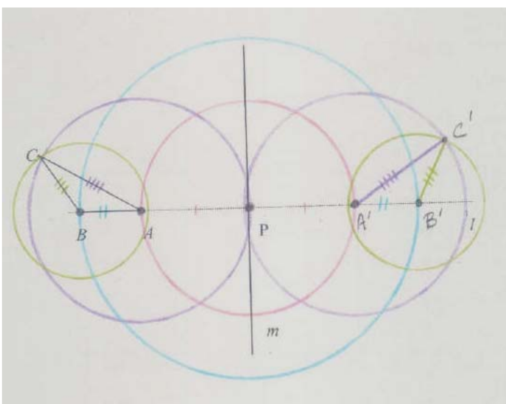
The following is one possible method that a student could choose to develop this construction.





A circle with a radius of $\overline{AP}$ was drawn with a center at P. A second circle was drawn with a radius of $\overline{BP}$ and a center at P.

A circle was drawn with a center at B and a radius of $\overline{BC}$. A congruent circle was drawn with a center at B'.



Congruent circles: one circle with the radius $\overline{AC}$ and one circle with the radius $\overline{A'C'}$ were drawn with center at A and A'. The construction is completed upon drawing the radius $\overline{B'C'}$ and the radius $\overline{A'C'}$.