Functions, Relations

Students will:
- Draw Venn diagrams with 2 or 3 circles and use them to compute set and subset membership and probabilities of membership.
- Identify a correspondence between variables as a relation and determine if the relation is a function.
- Translate among multiple representations of functions and relations.
- Describe an arithmetic sequence as an example of a linear function and identify specific characteristics shared by arithmetic sequences and linear functions.

Classroom Cases:
1. A researcher interviewed a group of 55 students about the movies that they had seen recently. She determined the following:
   - 17 had seen an “action” movie
   - 23 had seen a “comedy”
   - 6 had seen an “action” movie and an “adventure” movie
   - 10 had seen an “adventure” movie and a “comedy”
   - 2 had seen all of these movie genres

   How many students had seen exactly 2 of these movie genres? How many students had seen none of these movie genres? What is the probability that a randomly chosen student had seen exactly one of these movie genres?

   Case Closed - Evidence:
   In the intersections of each pair of circles, there are 4, 6, and 8 students for a total of 18 students who saw exactly 2 of the movies.
   There are 35 students in the circles. 55-35=20. There are 20 students who had not seen any of these movies. Out of the 55 students surveyed, 15 (5+3+7) had seen exactly one of these movies. The probability of randomly choosing a student who had seen exactly one of these movies is 15/55 = 3/11.

   2. A car whose original value was $26500 decreases in value $80 per month.
   a. Make a table for the value of the car during its first six months.
   b. Write a recursive function to represent the sequence in your table.
   c. Write an explicit formula to represent the sequence.
   d. Does the sequence represent a function?
   e. Predict the value of the car at the end of two years.
   f. When will the car’s value be $21000?

   Case Closed - Evidence:
   \[ V_n = 26500 - 80(n-1) \]
   a. \[ V_1 = 26500 \]
   b. \[ V_2 = V_0 - 80 \]
   c. \[ V = 26500 - 80(n-1) \]
   d. The sequence represents a linear function because it has a constant rate of change.
   e. \[ V = 26500 - 80(24 - 1) = 24660 \] The value of the car will be $24660.
   f. \[ 21000 = 26500 - 80(n-1) \] In 69.75 months, the car’s value will be $21000.

   3. Which of the following relations are also functions?
   a. \{ (1,2), (2,3), (1,4), (4,1) \}  b. \[ y = 3x + 22 \]
   c. \{ senators, states \}  d. \{ states, senators \}
   e. \{ states, senators \}  f. Not a function; fails vertical line test.
   g. Function because every input value corresponds to just one output value.

   Case Closed - Evidence:
   a. Not a function because 1 has 2 different outputs.
   b. Linear function
   c. Non-linear function
   d. Function; every senator represents one state.
   e. Not a function; every state has more than one senator.
   f. Not a function; fails vertical line test.
   g. Function because every input value corresponds to just one output value.

Terminology:
- Complement of a set: The collection of all items not in the set
- Element: A member or item in a set
- Explicit form: An algebraic expression that produces terms of a sequence by substitution of the term numbers.
- Function: A rule for matching elements of two sets in which an element from the first set matches only one element in the second set.
- Intersection: The set of all elements contained in all the given sets.
- Null set: A set that contains no elements. Also called an “empty set”.
- Recursive form: A set of algebraic expressions that produce the next term in a sequence.
- Relation: A rule that gives an output value for every valid input.
- Set: A collection of numbers, geometric figures, letters, or other objects that have some common characteristic.
- Subset: A collection of items drawn entirely from a single set. A subset can consist of any number of items from a set, ranging from none (null set) to the entire set.
- Union: The set of all elements that belong to at least one of the given sets.

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Further investigations:
Have a contest to see who can identify the most arithmetic sequences. Consider license plates, bar codes, zip codes, telephone numbers, grocery receipts, or any other objects that have sequences.

List pairs of objects or relationships such as (husbands, wives), (customs, stores) or (mayors, cities). Do the pairs form functions?

Make up puzzles like Classroom Case 1. Draw Venn diagrams to solve the puzzles.

You have many sets around your home, such as sets of dishes, sets of silverware, swing sets, paint sets, sets of tools. Discuss with your student why these are called ‘sets’. What would the union of a set of dishes and a set of silverware include? How would you describe the intersection of a set of tools and a set of glasses? If you have art supplies, what would be in the complement of your paint set?