Further investigations:
Invite your student to help you make dinner. What two dimensional shapes can he make by slicing a cucumber, a carrot, a block or wedge of cheese, and jellied cranberry sauce?

Ask your student to identify 3-D objects in the neighborhood. Ask her how the objects could be built by stacking 2-D shapes. Then discuss what the cross-section would be if the object were sliced by a plane. For example, a chimney may be a right rectangular prism. It could be built by stacking congruent rectangles. When the chimney is sliced by the plane of the roof, the cross section is a parallelogram. Think about a church steeple, a fire hydrant, and dormers.

Play an advanced version of “I Spy.” The objects spied must be described as plane figures translated or rotated through space. For example, a wastebasket might be a trapezoid with bases of 8 inches and 12 inches and height of 14 inches rotated about its midline.

Terminology:
Cross section: A plane figure obtained by slicing a solid with a plane.

Cylinder: A three-dimensional object with two parallel congruent circular bases.

Lateral faces of a pyramid: Faces that intersect at the vertex.

Lateral faces of prism: Faces that are not the bases of the solid.

Oblique figure: Prisms and cylinders with bases that are not aligned one directly above the other. Pyramids and cones with apex that are not aligned above the center of the base.

Polyhedron: A collection of polygons joined at their edges. Each of these polygons is called a “face.”

Prism: A polyhedron with two parallel and congruent faces and all other faces that are parallelograms.

Right figure: Prisms and cylinders with bases that are aligned one directly above the other. Pyramids and cones in which the apex lies on the perpendicular line that passes through the center of the base.

Related Files:
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Slices and Shadows

Students will:  Seventh Grade 7 of 7

- Create three-dimensional (3-D) objects by translating (sliding) and/or rotating (turning) two-dimensional plane figures
- Explore two-dimensional (2-D) cross sections of cylinders, cones, pyramids, and prisms

Classroom Cases:
1. Construct a rectangle that is not a square from cardboard. Label the vertices in order A, B, C, D. Cut out the rectangle and rotate it 360° about side AD.
   a. What shape have you formed?
   b. What is the volume of your shape?
   c. What is the surface area?

Case Closed - Evidence:

a. The shape I form is a cylinder. It has 2 circular bases. b. The volume tells how much space the cylinder occupies. It can be calculated by multiplying the area of one of the bases times the height.

V = \pi r^2 h = 3.14 \cdot 3^2 \cdot 8 = 223.2 \text{ cm}^3.

2. Lay the rectangle from case 1 on a flat surface and translate it vertically or diagonally, but not horizontally.
   a. What shape have you formed?
   b. What is the volume of your shape?
   c. What is the surface area?
   d. Are the volume and surface area in this example the same as they were in case 1? Please explain.
   e. How would translating in a different direction change the solid?

Case Closed - Evidence:

a. I formed a right rectangular prism. It has six rectangular faces. Opposite pairs of faces are parallel and congruent rectangles.
   b. I translated the original rectangle 6 cm. My prism occupies lh = 8 \cdot 3 \cdot 6 = 144 \text{ cm}^3.
   c. I will have to cover 2 faces that are 3 \cdot 8 cm and 2 faces that are 3 \cdot 6 cm and 2 faces that are 6 \cdot 8 cm. SA = 2 \cdot 3 \cdot 8 + 2 \cdot 3 \cdot 6 + 2 \cdot 6 \cdot 8 = 48 + 36 + 96 = 180 \text{ cm}^2.
   d. The volumes and surface areas are not the same. Although both solids started with the same rectangle, the transformations have created different shapes and these shapes have different dimensions that lead to different volumes and surface areas.
   e. If I translated a rectangle through space perpendicular to the plane containing the original rectangle, I would get a right rectangular prism. If I translated the rectangle in a direction that is not perpendicular to the original plane, I would get an oblique rectangular prism. Its bases are rectangles and its lateral faces are non-rectangular parallelograms.

3. Make a cone from modeling clay. Use dental floss to make slices.
   a. List the 2-D shapes you can make and some that you cannot make with single slices.
   b. Make a slice parallel to the base halfway between the base and the vertex. Compare the top shape with your original cone.

Case Closed - Evidence:

a. From a cone, I can make circles and ellipses. I cannot make any polygons because polygons have straight sides and cones are curved.
   b. My original cone and the top half of my slice are similar cones. The cross section is a circle with a radius 1/2 the length of the radius of the base of the original cone.